Republic of Iraq

Ministry of Higher Education and Scientific
University of Baghdad College of Education for Pure Sciences \ Ibn Al-Haitham

Soft Strongly Generalized Closed Set with respect to an Ideal in Soft Topological Space

A Thesis

Submitted to the College of Education Pure Sciences Ibn Al-Haitham, University of Baghdad as a partial Fulfillment of the Requirements for the Degree of Master of science in Mathematics.

By

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March, 2015 *Jumada al-awwal 1436*

بسم الله الرحمن الرحيم

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صدق الله العظيم (سورة يوسف ايه 76)

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We certify that this thesis was prepared under my supervision at the University, College of Education for Pure Sciences \ Ibn Al-Haitham, Department of Mathematics, as a partial fulfillment of the Requirements for the degree of Master of Science in Mathematics.

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DEDICATION

To whom gave me their tenderness, my parents

ACKNOWLEDGEMENT

Thanks to my God Allah Who takes the care of all works I do through his mercy and benignity.

I am most grateful to my supervisor Assist. Prof. Dr. Narjis Abduljabbar for her advice, guidance, helpful suggestions and encouragement throughout period I have worked under her supervision. I also wish to express my thanks to my colleague Saja with her family and the staff of the Department of Mathematics.

Alyasaa

March, 2015

TABLE OF NOTATIONS

(X,τ,E,I)	Soft topological space with an ideal I.
SSIg-closed	Soft strongly generalized closed set with respect to an ideal *I*.
SSIg-open	Soft strongly generalized open set with respect to an ideal *I*.
$cl^*(A,E)$	is a Soft strongly generalized closure set with respect to an ideal of (A,E).
$int^*(A,E)$	is a Soft strongly generalized interior set with respect to an ideal of (A,E).
$b^*(A,E)$	is a Soft strongly generalized border set with respect to an ideal of (A,E).
$bd^*(A,E)$	is a Soft strongly generalized boundary set with respect to an ideal of (A,E).
$\dot{D}(A,E)$	is a Soft strongly generalized derived set with respect to an ideal of (A,E).
f_{pu}	is a Soft mapping.
SSIg- Continuous	is a Soft strongly generalized continuous mapping with respect to an ideal *I*.
SSIg- irresolute	is a Soft strongly generalized irresolute mapping with respect to an ideal *I*.

Abstract

In this work, we introduced and studied a new kind of soft generalized closed set in soft topological spaces with an ideal, which we called soft strongly generalized closed set with respect to an ideal where a soft subset *(A,E)* of a soft topological space with an ideal *I*, (X,τ,E) is said to be soft strongly generalized closed set with respect to an ideal *I* ,(briefly SSIg- closed), if $cl(int(A,E))-(B,E)\in I$, whenever $(A,E)\tilde{\subseteq}(B,E)$ and *(B,E)* is soft open set. And denoted by SSIg-closed set . The complement of SSIg-closed set is called an SSIg-open set.

We studied the properties of SSIg-closed set, then we used SSIg-open set to define five kinds of derived sets, which are the SSIg-interior, SSIg-closure, SSIg-derived, SSIg-border, and SSIg-boundary with their relations and properties .

On the other side, we define new kinds of soft mappings between soft topological spaces, like SSIg-continuous, Contra-SSIg-continuous, SSIg-open, SSIg-closed and SSIg-irresolute mapping we studied the relations between these kinds of mappings and the composition of two mappings of the same type of two different types, with proofs or counter examples.

Table of contents

Introduction

In (1970) Levine[8] introduced the concept of generalized closed (g-closed) sets. From that year many authors used this notion to define other kinds of weakly or strongly types of this set, or used it to prove many facts in general topology.

The notion of ideal topological space was introduced in (1967). In recent years ideals become a very important tool in topology, so many researchers worked in this field like Hamlet T. R. and Jankovic D. (1988)[4] .

Soft set theory was first introduced by Molodtstov in (1999) [10], as a generalization of fuzzy sets, soft sets are used as a tool to ideal with uncertain objects. Recently, in (2014), the study of soft topological spaces was introduced by Georgiou D. N., Megaritis A. C. [1], they used the concept of soft set to define a topology, that leads to a new world in general topology.

The above concepts are used in this work, to define a new soft set in soft topological space, called soft Strongly generalized closed set with respect to an ideal in soft topological space , and is denoted by SSIg-closed set .

This thesis consists of three chapters. Chapter one contains three sections, we review the definitions of generalized closed and strongly generalized closed sets, with their properties. We also found the collection of these sets in some known spaces. In section two we give the preliminaries of Ig-closed sets. In section three, we summarize the notion of soft set with details about these sets, namely, intersection, complement and product. The notions of absolute and null sets are also given, with some examples. The concept of soft generalized closed set is reviewed in this section with examples and properties.

Chapter two contains six sections. In section one, we give the concept of strongly generalized closed set with respect to an ideal (SSIg-closed). We proved many theorems and give many examples to explain some facts or disprove others, with details. In section two, we give the notion of SSIg-interior of a soft set, with some facts and examples. In section three the concept of SSIg-closure was introduced with details. In section four we defined the SSIg-derived set with properties and examples. In section five, we present the concept of SSIg-border. In section six we define SSIg-boundary of a soft set with some properties.

Chapter three consists of three sections. In section one, we review some definitions of g-mappings and g-homeomorphisms. In section two, the concepts of soft mapping, soft continuous, soft open, soft closed and soft homeomorphism mappings are introduced. In section three, we give the definitions of SSIg-continuous, SSIg-open, SSIg-closed, SSIg-irresolute, Contra-SSIg-continuous and SSIg-homeomorphisms. The composition of these mappings are also discussed.

In this Chapter , we review three different branches in general topology, in order to mixed them in Chapters two and three. The first branch is the concept of generalized and strongly generalized closed sets. The second is the concept of ideal on a topological space with the meaning of space with ideal. The third branch is the definition of soft set in mathematics, then the soft set in soft topological space. "The interior and the closure of a subset *A* of a topological space (X,τ) are denoted by *int(A)* and *cl(A)*, respectively."[19]

1.1 Generalized closed sets and strongly generalized closed sets.

"***Definition(1.1.1) :***

Let *A* and *B* be two nonempty subsets in a topological space (X,τ). Then *A* and *B* are said to be ***separated*** if $cl(A)\cap B=\phi$ and $A\cap cl(B)=\phi$."[19]

"***Definition(1.1.2):***

Let X be a topological space . A subset *A* of X is said to be ***generalized closed*** (briefly, g-closed) set if $cl(A)\subseteq U$, whenever $A\subseteq U$ and *U* is an open set.

The complement of a generalized closed set is called a ***generalized open*** (briefly, g-open) set. "[15]

"***Remark(1.1.3):***

i- Every closed set in a topological space (X,τ) is a generalized closed set.
ii- Every open set in a topological space (X,τ) is a generalized open set. "[9]

"***Theorem(1.1.4):***

A set *A* is a g-closed set if and only if *cl(A)-A* contains no nonempty closed subset. "[9]

"***Corollary(1.1.5) :***

A g-closed set A is closed if and only if $cl(A)-A$ is a closed set. "[9]

"***Theorem(1.1.6):***

If A and B are g-closed sets, then $A \cup B$ is an g-closed set. "[9]

"***Remark(1.1.7):***

The intersection of two g-closed sets in general is not a g-closed set . "[9]

"***Proposition(1.1.8) :***

Let A be a g-closed set and suppose that F is a closed set. Then $A \cap F$ is a g-closed set. "[15]

"***Corollary(1.1.9) :***

If A and B are separated g-open sets, then $A \cup B$ is g-open. "[9]

"***Theorem (1.1.10):***

Suppose that $B \subseteq A \subseteq X$, B is a g-closed set relative to A and that A is a g-closed subset of X. Then B is g-closed relative to X. "[9]

"***Proposition(1.1.11):***

If A is a g-closed set and $A \subseteq B \subseteq cl(A)$, then B is a g-closed set. "[9]

"***Proposition(1.1.12):***

If $\text{int}(A) \subseteq B \subseteq A$ and if A is g-open, then B is g-open. "[9]

"***Proposition(1.1.13):***

Let $A \subseteq Y \subseteq X$ and suppose that A is g-closed set in X. Then A is g-closed set relative to Y. "[9]

"***Proposition(1.1.14):***

If $A \subseteq Y \subseteq X$ where A is g-open set relative to Y and Y is g-open set relative to X, then A is g-open relative to X. "[9]

"***Theorem(1.1.15):***

In a topological space (X,τ), $\tau = \xi$ (the family of all closed subsets of X) if and only if every subset of X is a g-closed set."[9]

"***Theorem(1.1.16):***

A set A is g-open if and only if $F \subseteq \text{int}(A)$ whenever F is closed set and $F \subseteq A$."[9]

"***Definition(1.1.17)***:

For the subset A of a topological space X, the generalized closure operator $cl_g(A)$ is defined as the intersection of all g-closed sets containing A. "[15]

"***Proposition(1.1.18) :***

Let A and B be two g-closed sets and suppose that A^c and B^c are separated. Then $A \cap B$ is g-closed. "[9]

"***Theorem(1.1.19) :***

A set A is g-open in (X,τ) if and only if $B = X$ whenever B is open and $\text{int}(A) \cup A^c \subseteq B$."[9]

"***Theorem(1.1.20):***

Let A be a subset of a topological space (X,τ). Then A is g-closed if and only if $cl(A) - A$ is an g-open set. "[9]

"***Definition(1.1.21):***

Let (X,τ) be a topological space and A be a subset of X , then A is ***strongly generalized closed set*** (briefly sg-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an open set. The complement of an sg-closed set is called a Sg-open set. We denote the family of all strongly generalized closed sets by SGC(X) and the family of all strongly generalized open sets by SGO(X). "[15]

"***Theorem (1.1.22) :***

Let (X,τ) be a topological space. Then every generalized closed set is strongly generalized closed set. "[9]

"***Corollary(1.1.23):***

Every closed set in a topological space (X,τ) is a strongly generalized closed set. "[9]

"***Definition(1.1.24) :***

Let A be any set in a topological space (X,τ). The ***border*** of a set A denoted by bA is defined as bA= A-*intA*. "[19]

1.2 Generalized and strongly generalized closed set with respect to an ideal I .

"***Definition(1.2.1)*:**

An ideal on a set X is a nonempty collection I of subsets of X with heredity property and finite additivity property, that is, it satisfies the following two conditions:

1. $A \in I$ and $B \subseteq A$ then $B \in I$ (heredity),
2. $A \in I$ and $B \in I$ implies $A \cup B \in I$ (finite additivity).

We denote a topological space (X,τ) with an ideal I defined on X by (X,τ,I).

If I is an ideal on X and Y is a subset of X, then $I_Y = \{Y \cap I_\alpha : I_\alpha \in I, \alpha \in \Lambda\}$ is an ideal on Y."[15]

"***Definition(1.2.2)*:**

Let (X,τ) be a topological space and I be an ideal on X. A subset A of X is said to be ***generalized closed set with respect to an ideal I*** (briefly Ig-closed) if $cl(A)-B \in I$, whenever $A \subseteq B$ and B is an open set. The complement of an Ig-closed set is called a ***generalized open set with respect to an ideal I*** (briefly Ig-open). We denote the family of all generalized closed sets with respect to an ideal I by IGC(X) and the family of all generalized open sets with respect to an ideal I by IGO(X)."[5]

"***Proposition(1.2.3)***:

Let (X,τ) be a topological space with an ideal *I*. Then every g-closed subset is Ig-closed. "[5]

"***Remark(1.2.4) :***

The converse of Proposition*(1.2.3)* need not be true. "[5]

"***Theorem(1.2.5):***

A set *A* is Ig-closed in (X,τ) if and only if $F \subseteq cl(A)-A$ and F is closed set in X implies $F \in I$. "[5]

"***Theorem(1.2.6):***

If *A* and *B* are Ig-closed sets in (X,τ), then $A \cup B$ is an Ig-closed set. "[5]

"***Remark (1.2.7):***

The intersection of two Ig-closed sets need not be an Ig-closed ."[5]

Remark (1.2.8):

The infinite union of Ig-closed sets need not be an Ig-closed by the following example.

Example :

Let (R,τ_u) be the usual topological space. Let $H_n = \left[\frac{1}{n}, 1\right]$ for $n \geq 2$. Suppose that *A* is an open set such that $H_n \subseteq A$, $n \geq 2$, then $cl(H_n) - A = H_n$ - $A = \phi$, $n \geq 2$. Hence $cl(H_n)$ -A=ϕ. Thus H_n is Ig-closed for each $n \geq 2$. But if we consider $I = \{\phi\}$ and $A = (0,2)$, then $\bigcup_{n\geq 2} H_n = (0,1] \subseteq (0,2)$. But $cl\{\bigcup_{n\geq 2} H_n\} - (0,2) = [0,1] - (0,2) = \{0\} \notin I$. Thus $\bigcup_{n\geq 2} H_n$ is not Ig-closed.

"***Theorem (1.2.10):***

If *A* is Ig-closed set and $A \subset B \subset cl(A)$ in (X,τ), then *B* is Ig-closed set in (X,τ) . "[5]

"***Theorem(1.2.11) :***

If *int(A)* $\subseteq B \subseteq A$ and if *A* is Ig-open set in (X,τ), then *B* is Ig-open set in X. "[5]

"***Theorem (1.2.12) :***

Let $A \subset Y \subset X$ and suppose that *A* is Ig-closed set in (X,τ). Then *A* is Ig-closed set relative to the subspace Y of X, with respect to the ideal I_Y. "[5]

"***Theorem(1.2.13):***

If $A \subseteq B \subseteq X$, *A* is Ig-open set relative to *B* and *B* is Ig-open set relative to X, then *A* is Ig-open set relative to X. "[5]

"***Theorem(1.2.14) :***

Let *A* be an Ig-closed set and a closed set *F* in (X,τ) . Then $A \cap F$ is an Ig-closed set in (X,τ) . "[5]

"***Theorem(1.2.15):***

A set *A* is Ig-open set in (X,τ) if and only if $F - U \subseteq int(A)$, for some $U \in I$, whenever $F \subseteq A$ and *F* is closed set. "[5]

"***Theorem(1.2.16):***

If *A* and *B* are separated Ig-open sets in (X,τ), then $A \cup B$ is Ig-open set. "[5]

"***Corollary(1.2.17):***

Let *A* and *B* be Ig-closed sets. And suppose $X - A$ and $X - B$ are separated in (X,τ). Then $A \cap B$ is Ig-closed. "[5]

"***Theorem(1.2.18):***

A set *A* is Ig-closed set in (X,τ), if and only if $cl(A) - A$ is Ig-open set. "[5]

"***Definition(1.2.19) :***

Let (X, τ) be a topological space and I be an ideal on X . A subset A of X is said to be ***strongly generalized closed set with respect to an ideal*** (briefly SIg- closed) if $cl(int(A))-B \in I$ whenever $A \subseteq B$ and B is open set. The complement of an SIg-closed set is called ***a strongly generalized open set with respect to an ideal*** (briefly SIg- open). We denote the family of all strongly generalized closed sets with respect to an ideal I by ISGC(X) and the family of all strongly generalized open sets with respect to an ideal I by ISGO(X). "[15]

"***Theorem(1.2.20) :***

Every g- closed set is an SIg-closed set. "[15]

"***Remark(1.2.21):***

The converse of Theorem(1.2.20) need not be true. "[15]

"***Remark(1.2.22):***

The intersection of two SIg-closed sets need not be an SIg-closed set. "[15]

"***Theorem(1.2.23) :***

If A and B are two elements in ISGC(X), then their union also is a element in ISGC(X). "[15]

"***Corollary(1.2.24):***

If A and B are SIg-open sets in (X,τ) , then $A \cap B$ is SIg-open set. "[5]

Remark (1.2.25):

In the following example we show that the infinite union of SIg-closed sets need not be an SIg-closed set.

Example :

Let (R,τ_u) be the usual topological space. Let $H_n = \left[\frac{1}{n},1\right]$ for $n \geq 2$. Suppose that A is an open set such that $H_n \subseteq A$, $n \geq 2$, then $cl(\text{int}(H_n)) - A = H_n$-$A$=$\phi, n \geq 2$. Hence $cl(\text{int}(H_n))$-A=ϕ. Thus H_n is SIg-closed for each $n \geq 2$. But if we consider I={ϕ} and A= $(0,2)$, then $\underset{n\geq 2}{\cup} H_n$ $=(0,1] \subseteq (0,2)$. But $cl(\text{int}(\{\underset{n\geq 2}{\cup} H_n\}) - (0,2) = [0,1] - (0,2) = \{0\} \notin I$. Thus $\underset{n\geq 2}{\cup} H_n$ is not SIg-closed.

"***Theorem (1.2.26) :***

The intersection of SIg-closed set and a closed set F in (X, τ) is an SIg-closed set in (X, τ). "[15]

1.3 Soft set .

" To avoid difficulties, one must use an adequate parametrization . Let X be an initial universe set and let E be a set of parameters."[13]

"***Definition (1.3.1):***

For $A \subseteq E$, the pair (F,A) is called a ***soft set*** over X, where F is a mapping given by $F{:}A \to P(X)$.

In other words, the soft set is a parametrized family of subsets of the set X. Every set $F(e)$, $e \in E$, from this family may be considered as the set of e-elements of the soft set (F,E) , or as the set of e-approximate elements of the soft set. A pair (a,A) is said to be a soft point where a(e)$\neq \phi$, $\forall e \in A$ and a(e') $=\phi$, $\forall e' \in A - \{e\}$. Clearly, a soft set is not a set."[2]

"***Remark(1.3.2):***

As an illustration, let us consider the following examples. But before we give the example, we need the following definition."[2]

"***Definition (1.3.3) :***

Let X be an initial universe set and let E be a set of parameters . Every primitive attribute a∈E is a total mapping a: E →V_a where V_a is the set of values of a, called domain of a. With every subset of attributes B⊆E, we associate a binary relation IND(B), called an indiscernibility relation, defined by IND(B)={(x,y)∈X^2, for every a∈B , a(x)=a(y) }."[13]

"***Definition (1.3.4) :***

Let *R* be a family of equivalence relations and let $A \in R$. We say that *A* is dispensable in *R* if IND(*R*) = IND(*R*-{*A*}); otherwise *A* is indispensable in *R*. The family *R* is independent if each $A \in R$ is indispensable in *R*; otherwise *R* is dependent. $Q \subseteq P$ is a reduction of *P* if *Q* is independent and IND(*Q*) = IND(*P*), that is to say *Q* is the minimal subset of *P* that keeps the classification ability. The set of all indispensable relations in *P* will be called the core of *P*, and will be denoted as CORE(*P*). Clearly, CORE(*P*) =∩RRED(*P*), where RED(*P*) is the family of all reductions of *P*. "[13]

"***Example (1.3.5) :***

Let *U*= $\{h_1,h_2,h_3,h_4,h_5,h_6\}$ be a set of six houses, E = {expensive; beautiful; wooden; cheap; in green surroundings; modern; in good repair; in bad repair}, be a set of parameters.

Consider the soft set (F,E) which describes the 'attractiveness of the house', given by *(F,E)* = {expensive houses =ϕ, beautiful houses = $\{h_1,h_2,h_3,h_4,h_5,h_6\}$, wooden houses = $\{h_1,h_2,h_6\}$, modern houses = $\{h_1,h_2,h_6\}$, houses in bad repair = $\{h_2,h_4,h_5\}$, cheap houses = $\{h_1,h_2,h_3,h_4,h_5,h_6\}$, houses in good repair = $\{h_1, h_3, h_6\}$, houses in green surroundings = $\{h_1, h_2, h_3, h_4, h_6\}$}.

Suppose that, Mr. X is interested in buying a house on the basis of his choice parameters 'beautiful', 'wooden', 'cheap', 'in green surroundings', 'in good repair', etc., which constitute the subset *P* = {beautiful, wooden, cheap, in green surroundings, in good repair} of the set E. That means, out of available houses in *U*, he is to select that house which qualifies with all (or with maximum number of) parameters of the soft set *P*.

U	e_1	e_2	e_3	e_4	e_5
h_1	1	1	1	1	1
h_2	1	1	1	1	0
h_3	1	0	1	1	1
h_4	1	0	1	1	0
h_5	1	0	1	0	0
h_6	1	1	1	1	1

Table1

To solve this problem, the soft set (F,P) is firstly expressed as a binary table as shown above.

If $h_i \in F(e_j)$ then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries in Table1.

Thus, a soft set can now be viewed as a knowledge representation system where the set of attributes is replaced by a set of parameters.

Consider the tabular representation of the soft set (F,P). If Q is a reduction of P, then the soft set (F,Q) is called the reduct-soft-set of the soft set (F, P).

The choice value of an object $h_i \in U$ is c_i, given by $c_i = \sum_j h_{ij}$, where h_{ij} are the entries in the table of the reduct-soft-set.

The algorithm for Mr. X to select the house he wishes is listed as follows.

1. Input the soft set (F,E),
2. Input the set P of choice parameters of Mr. X which is a subset of E,
3. Find all reduct-soft-sets of (F, P),
4. Choose one reduct-soft-set say (F, Q) of (F, P),
5. Find k, for which $C_k = \max c_i$.

Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X using his option.

We claimed that $\{e_1, e_2, e_4, e_5\}$ and $\{e_2, e_3, e_4, e_5\}$ are two reductions of $P = \{e_1, e_2, e_3, e_4, e_5\}$. But $\{e_1, e_2, e_4, e_5\}$ and $\{e_2, e_3, e_4, e_5\}$ are not really the reductions of $P = \{e_1, e_2, e_3, e_4, e_5\}$.

Our following computing results will illustrate this.

Suppose R_p is the indiscernibility relation induced by $P = \{e_1, e_2, e_3, e_4, e_5\}$, then the partition defined by R_p is $(\{h_1, h_6\}, \{h_2\}, \{h_3\}, \{h_4\}, \{h_5\})$. If we delete $\{e_1, e_3\}$ from P, then the indiscernibility relation and the partition are invariant, so both of e_1 and e_3 are dispensable in P. If we delete one of $\{e_2, e_4, e_5\}$ from P, then the indiscernibility relation and the partition would be changed, thus all of these three parameters are indispensable. For example, suppose we delete $\{e_2\}$ from P, then the partition is changed to $(\{h_1, h_3, h_6\}, \{h_2, h_4\}, \{h_5\})$. So $\{e_2, e_4, e_5\}$ is in fact the reduction of $P = \{e_1, e_2, e_3, e_4, e_5\}$.

From Table1 we can also conclude that e_1 and e_3 are not relevant and will not affect the choices of the house since they take the same values for every house.

On the other hand, in this algorithm they compute the reduction of the soft set in step 3 before computing the choice value in step 5, which would lead to two problems. First, after reduction, the objects that take max choice value may be changed, so it is possible that the decision after reduction is not the best one. Second, since the reductions of soft set are not unique, it is possible that there would be a difference between the objects that take max choice value obtained using different reductions. In these two cases, the choice object may not be optimal or may be quite difficult to select. "[10]

"***Definition(1.3.6):***

A ***semi-ring*** is a nonempty set S equipped with two binary operations + and $*$, called addition and multiplication, such that:

1. $(S, +)$ is a commutative monoid with identity element 0:
    1. $(a + b) + c = a + (b + c)$,
    2. $0 + a = a + 0 = a$,
    3. $a + b = b + a$,
2. $(S, *)$ is a monoid with identity element 1:
    1. $(a*b)*c = a*(b*c)$,
    2. $1*a = a*1 = a$,
3. Multiplication left and right distributes over addition:
    1. $a*(b + c) = (a*b) + (a*c)$,
    2. $(a + b)*c = (a*c) + (b*c)$,
4. Multiplication by 0 annihilates S:
    1. $0*a = a*0 = 0$. "[22]

"***Definition(1.3.7):***

Let S be a semi-ring and A be a nonempty set. ρ will refer to an arbitrary binary relation between an element of A and an element of S, that is,

ρ is a subset of A×S without otherwise specified. A set-valued function $\eta: A \to P(S)$ can be defined as $\eta(x) = \{y \in S \mid (x,y) \in \rho\}$ for all $x \in A$. The pair (η, A) is a soft set over S, which is derived from the relation ρ."[22]

"***Example(1.3.8) :***

Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ be the semi-ring of integers module 6. Let (η, A) be a soft set over Z_6, where $A = Z_6$ and $\eta : A \to P(Z_6)$ is a set-valued function defined by $\eta(x) = \{y \in Z_6 ;\ x\ \rho\ y \Leftrightarrow xy \in \{0, 2, 4\}\}$ for all $x \in A$. Then $\eta(0) = Z_6$, $\eta(1) = \{0, 2, 4\}$, $\eta(2) = Z_6$, $\eta(3) = \{0, 2, 4\}$, $\eta(4) = Z_6$ and $\eta(5) = \{0, 2, 4\}$ are subsemi-rings of Z_6. Hence (η, A) is a soft semi-ring over Z_6. "[22]

"***Definition(1.3.9):***

A graph $G = (V,E)$ consists of a non-empty set of objects V, called vertices and a set E of two elements subsets of V called edges. Two vertices x and y are adjacent if $\{x, y\} \in E$. A graph $G = (V',E')$ is said to be a subgraph of $G = (V,E)$ if $V' \subseteq V$ and $E' \subseteq E$. For any subset S of the vertex set of the graph G, the induced subgraph S' is the subgraph of G whose vertex set is S and two vertices are adjacent in S if and only if they are adjacent in G. A graph G is called a ***simple graph*** is an undirected graph that has no loops (edges connected at both ends to the same vertex) and no more than one edge between any two different vertices. Also, it is called **connected** if every pair of vertices in the graph is connected.

Let $G = (V,E)$ be a simple graph, A any nonempty set. Let R an arbitrary relation between elements of A and elements of V. That is $R \subseteq A \times V$. A set valued mapping $F : A \to P(V)$ can be defined as $F(x) = \{y \in V;\ xRy\}$. The pair (F, A) is a soft set over V. "[18]

"***Definition(1.3.10) :***

Let (F, A) be a soft set over V. Then (F, A) is said to be a soft graph of G if the subgraph induced by $F(x)$ in G, $F'(x)$ is a connected subgraph of G for all $x \in A$. "[18]

"***Example(1.3.11) :***

Consider the graph $G = (V,E)$ as shown in Fig.1. Let $A = \{v_1 , v_3 , v_5\}$. Define the set valued mapping F by, $F(x) = \{y \in V;\ xRy \Leftrightarrow x \text{ is adjacent to } y \text{ in } G\}$. Then $F(v_1) = \{v_2 , v_5\}$, $F(v_3) = \{v_2 , v_4\}$, $F(v_5) = \{v_1 , v_2 , v_4\}$.
Here subgraph induced by $F(x)$ in G, $F'(x)$ is a connected subgraph of G, for all $x \in A$.

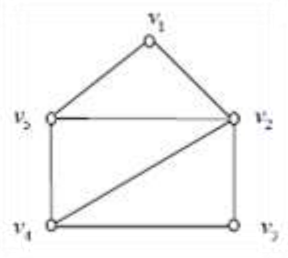


Fig.1 "[18]

"***Example(1.3.12) :***

Zadeh's fuzzy set may be considered as a special case of the soft set. Let A be a fuzzy set , and μ_A be the membership function of the fuzzy set A, that is μ_A is a mapping of U into [0, 1] .

Let us consider the family of α -level sets for function μ_A

$$F(\alpha)=\{x \in U\ ;\mu_A(x) \geq \alpha\}\ ,\ \alpha \in [0,1]$$

If we know the family F, we can find the functions $\mu_A(x)$ by means of the following formulae:

$$\mu_A(x)=\sup\{\ \alpha\ ;\ \alpha \in [0,1]\ ,x \in F(\alpha)\ \}$$

Thus, every Zadeh's fuzzy set A may be considered as the soft set $(F, [0,1])$. "[13]

"***Note(1.3.13):***

In what follows by *SS(X,E)* we denote the family of all soft sets over X. "[1]

"***Definition(1.3.14):***

Assume that we have a binary operation, denoted by $*$, for subsets of the set X. Let (F,*A*) and (G,B) be soft sets over X. Then, the operation $*$ for soft sets is defined in the following way :

$(F, A)*(G, B) = (H, A\times B)$, where $H(\alpha,\beta)=F(\alpha)*G(\beta)$, $\alpha\in A, \beta\in B$, and $A\times B$ is the cartesian product of the sets *A* and B .

This definition takes into account the individual nature of any soft set. "[10]

"***Definition(1.3.15):***

For two soft sets *(F,A)* and *(G,B)* in *SS(X,E)* , we say that *(F, A)* is a soft subset of *(G, B)* if $A\subseteq B$ and $F(e)\subseteq G(e), \forall e\in A$ **.**

Also, we say that the pairs *(F,A)* and *(G,B)* are soft equal if $(F,A)\,\tilde{\subseteq}\,(G,B)$ and $(G,B)\,\tilde{\subseteq}\,(F,A)$. Symbolically, we write *(F,A) = (G,B).*"[1]

"***Definition (1.3.16) :***

The union of two soft sets (*F*, *A*) and (*G*,*B*) over the common universe *X* is the soft set (*H*,*C*), where $C = A\cup B$ and for all $e\in C$,

$$H(e)=\begin{cases} F(e) & ,e\in A-B,\\ G(e) & ,e\in B-A,\\ F(e)\cup G(e) & ,e\in A\cap B\ . \end{cases}$$

"[7]

"***Definition(1.3.17):***

The intersection of two soft sets (*F*,*A*) and (*G*,*B*) over the common universe *X* is the soft set (*H*,*C*), where $C = A\cap B$ and for all $e\in C$, $H(e) = F(e)\cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are the same. "[16]

"***Definition (1.3.18) :***

The complement of a soft set (F,E), denoted by $(F,E)^c$ is defined by $(F,E)^c = (F^c, E)$, $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F,E)^c)^c = (F,E)$. "[16]

"***Definition(1.3.19) :***

The difference of two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E)-(G,E)$ is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) - G(e)$. "[3]

"***Definition(1.3.20) :***

Let (F,E) be a soft set over X and $x \in X$. We say that $x \tilde{\in} (F,E)$ read as x belongs to the soft set (F,E), whenever $x \in F(\alpha)$ for all $\alpha \in E$. Note that for $x \in X$, $x \tilde{\notin} (F,E)$ if $x \notin F(\alpha)$ for some $\alpha \in E$. "[3]

"***Definition(1.3.21):***

Let $(F,A) \in SS(X,A)$ and $(G,B) \in SS(X,B)$. The ***cartesian product*** of *(F,A)* and *(G,B)* is the soft set $(H, A \times B) \in SS(X \times Y, A \times B)$, where the map $H : A \times B \to P(X \times Y)$,such that $H(a,b) = F(a) \times F(b)$, for every $(a,b) \in A \times B$. Symbolically, we write $(H, A \times B) = (H,A) \tilde{\times} (H,B)$ and $H = F \times G$."[14]

"***Definition(1.3.22):***

A soft set (F,A) over X is said to be a ***null*** soft set, denoted by ϕ_A, if for all $e \in A$, $F(e) = \phi$ (null set), where $\phi_A(e) = \phi \quad \forall e \in A$."[10]

"***Definition(1.3.23) :***

A soft set (F,A) over X is said to be ***an absolute*** soft set, denoted by X_A, if for all $e \in A$, $F(e)= X$. Clearly, we have $X_A{}^c = \phi_A$ and $\phi_A{}^c = X_A$. "[16]

"***Definition(1.3.24) :***

Let Λ be an arbitrary indexed set and $L = \{(F_i,E),\ i \in \Lambda\}$ be a subfamily of $SS(X,E)$.

(1)The union of L is the soft set (H,E), where $H(e) = \bigcup_{i\in\Lambda} F_i(e)$ for each $e \in E$. We write $(H,E) = \tilde{\bigcup}_{i\in\Lambda}(F_i,E)$.

(2)The intersection of L is the soft set (M,E), where $M(e) = \bigcap_{i\in\Lambda} F_i(e)$ for each $e \in E$. We write $(M,E) = \tilde{\bigcap}_{i\in\Lambda}(F_i,E)$. "[1]

"***Proposition(1.3.25) :***

Let (F, A) and (G, B) be soft sets over X . Then

(1) $((F, A)\tilde{\cap}(G,B))^c = (F, A)^c \tilde{\cup} (G,B)^c$.

(2) $((F, A)\tilde{\cup}(G,B))^c = (F, A)^c \tilde{\cap} (G,B)^c$. "[12]

"***Proposition(1.3.26) :***

Let (F, A) be soft set over X . Then

(1) $(F, A)\tilde{\cap}(F, A) = (F, A)$.

(2) $(F, A)\tilde{\cup}(F, A) = (F, A)$.

(3) $(F, A)\tilde{\cap} X_A = (F, A)$

(4) $(F, A)\tilde{\cup} X_A = X_A$.

(5) $(F, A)\tilde{\cap} \phi_A = \phi_A$.

(6) $(F, A)\tilde{\cup} \phi_A = (F, A)$. "[12]

"***Definition(1.3.27) :***

Let τ be a collection of soft sets over X with the fixed set E of parameters and A⊆E, then τ⊆SS(X,E). We say that the family τ defines a soft topology on *X* if the following axioms are true :

(1) X_A, ϕ_A $\in \tau$,

(2) If (G,*A*) , (H,*A*) $\in \tau$, then (G,*A*)$\tilde{\cap}$(H,*A*) $\in \tau$,

(3) If $(G_i, A) \in \tau$ for every $i \in \Lambda$, then $\tilde{\cup}_{i \in \Lambda} (G_i, A) \in \tau$.

Then τ is called a **soft topology** on X and the triple (X,τ,E) is called soft topological spaces over X .
"[3]

"***Definition(1.3.28):***

Let(*X*,τ,*E*) be a soft topological space. The members of τ are said to be soft open sets in *X*. We denote the set of all soft open sets over *X* by *OS*(*X*,τ,*E*) or *OS*(*X*) and the set of all soft closed sets, which are the complements of soft open sets by *CS*(*X*,τ,*E*) or *CS*(*X*). "[6]

"***Definition (1.3.29) :***

Let (*X*,τ,*E*) be a soft topological space and *(F,E)*∈*SS(X,E)*. Define $\tau_{(F,E)}$ ={(*G*,*E*)$\tilde{\cap}$(*F*,*E*) : (*G*,*E*) ∈ τ }, which is a soft topology on (*F*,*E*). This soft topology is called soft relative topology on (*F*,*E*), then [(*F*,*E*), $\tau_{(F,E)}$] is called soft subspace of (*X*,τ,*E*). "[7]

"***Definition(1.3.30):***

A soft topological space with respect to an ideal I (*X*,τ,*E*) is called soft discrete topological space with respect to an ideal I if every soft subset in (*X*,τ,*E*) is soft open set . "[6]

"***Definition(1.3.31):***

Let (*X*, τ ,*E*) be a soft topological space and (*F*,*E*) ∈ *SS(X*,*E*). The soft closure of (*F*,*E*), denoted by *cl*(*F*,*E*) is the intersection of all closed soft super sets of (F,E) that to say $cl(F,E) = \tilde{\cap}\{(H,E)\ ;\ (H,E) \in CS(X)$ and $(F,E) \tilde{\subseteq} (H,E)\}$. "[16]

"**Definition (1.3.32) :**

Let (X, τ, E) be a soft topological space and $(G,E) \in SS(X,E)$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) that is to say $int(G,E) = \tilde{\cup}\{(H,E)\ ;\ (H,E) \in OS(X)$ and $(H,E) \tilde{\subseteq} (G,E)\}$. "[2]

"**Proposition(1.3.33) :**

Let (X, τ, E) be a soft topological space and (F,E) , $(H,E) \in SS(X,E)$. Then

(1) $cl(cl(F,E)=cl(F,E)$,

(2) $(F,E) \tilde{\subseteq} cl(F,E)$,

(3) $(F,E) \tilde{\subseteq} (H,E)$ *implies* $cl(F,E) \tilde{\subseteq} cl(H,E)$,

(4) $cl\{(F,E) \tilde{\cup} (H,E)\} = cl(F,E) \tilde{\cup} cl(H,E)$,

(5) $cl\{(F,E) \tilde{\cap} (H,E)\} \tilde{\supseteq} cl(F,E) \tilde{\cap} cl(H,E)$. "[3]

"**Proposition(1.3.34) :**

Let (X, τ, E) be a soft topological space and (F,E) , $(H,E) \in SS(X, E)$. Then

(1) $int(int(F,E)=int(F,E)$,

(2) $int(F,E) \tilde{\subseteq} (F,E)$,

(3) $(F,E) \tilde{\subseteq} (H,E)$ *implies* $int(F,E) \tilde{\subseteq} int(H,E)$,

(4) $int(F,E) \tilde{\cup} int(H,E) \tilde{\subseteq} int\{(F,E) \tilde{\cup} (H,E)\}$,

(5) $int\{(F,E) \tilde{\cap} (H,E)\} = int(F,E) \tilde{\cap} int(H,E)$. "[3]

Definition(1.3.35):

Let (A,E) be a soft set in a soft topological space (X,τ,E) with an ideal I, Then the ***soft border*** of (A,E) defined by $b(A,E) = (A,E) \tilde{\cap} cl(A,E)^c$ and denoted it by $b(A,E)$.

"***Definition(1.3.36):***

Two soft sets *(A,E)* and *(B,E)* are said to be ***soft separated*** in a soft topological space (X,τ,E) if $cl(A,E)\tilde{\cap}(B,E) = \phi_E$ and $(A,E)\tilde{\cap}cl(B,E) = \phi_E$."[11]

"***Definition(1.3.37) :***

Let *(A,E)* be a soft set over X and (X,τ,E) be soft topological space. Then the ***soft boundary*** of *(A,E)* denoted by bd*(A,E)* and defined as bd*(A,E)*= $cl(A,E)\tilde{\cap}cl(A,E)^c$. "[3]

"***Definition(1.3.38):***

A soft set *(A,E)* is called a ***soft generalized closed*** (soft g-closed) set in a soft topological space (X,τ,E) if $cl(A,E)\tilde{\subseteq}(U, E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and *(U,E)* is soft open set in X . The relative complement of *(A,E)* is called a ***soft generalized open*** (soft g-open) set. "[8]

"***Proposition(1.3.39) :***

Every soft closed set is soft generalized closed set."[8]

"***Corollary(1.3.40) :***

Every soft open set is soft generalized open set."[8]

"***Remark(1.3.41) :***

The converse of Proposition***(1.3**.*37)*** need not be true. "[8]

"***Theorem(1.3.42) :***

If *(A,E)* is soft g-closed set over X and $(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}cl(A,E)$, then *(B,E)* is soft g-closed set. "[1]

"***Theorem(1.3.43) :***

If (*A*,E) is soft g-open set over *X* and $int(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(A,E)$, then *(B,E)* is soft g-open set. "[8]

"***Theorem (1.3.44) :***

If *(A,E)* and *(B,E)* are soft g-closed sets, then so is $(A,E)\tilde{\cup}(B, E)$. "[8]

"***Corollary(1.3.45):***

If *(A,E)* and *(B,E)* are soft g-open sets, then so is $(A,E)\tilde{\cap}(B,E)$. "[8]

"***Theorem(1.3.46):***

A set *(A,E)* is soft g-closed set over *X* if and only if *cl(A,E)-(A,E)* contains only null soft closed set. "[8]

"***Theorem(1.3.47):***

A soft g-closed *(A,E)* is soft closed set if and only if *cl(A,E)-(A,E)* is soft closed set."[8]

"***Definition(1.3.48) :***

Let *E* be a set of parameters and $A,B\subseteq E$, A nonempty collections *I* of soft subsets over *X* is called a ***soft ideal*** on *X* if the following holds

(1) If $(F,A)\in I$ and $(G,B)\tilde{\subseteq}(F,A)$ implies $(G,B)\in I$ (heredity),

(2) If *(F,A)* and $(G,A)\in I$, then $(F,A)\tilde{\cup}(G,A)\in I$ (additivity).

If I is ideal on X and Y is subset of X, then $I_Y = \{Y_E \tilde{\cap} I_1 : I_1 \in I\}$ is an ideal on Y. We denoted for a soft topological space with respect to an ideal *I* by (X,τ,E,*I*) "[7]

"***Definition(1.3.49) :***

Let (X,τ,E) be a soft topological space with an ideal *I* . A soft set *(F,E)* $\in SS(X,E)$ is called ***soft generalized closed set with respect to an ideal I*** (soft Ig-closed) if $cl(F,E)-(G,E)\in I$ whenever $(F,E)\tilde{\subseteq}(G,E)$ and $(G,E)\in\tau$. The relative complement $(F,E)^c$ is called ***soft generalized open set with respect to an ideal I*** (soft Ig-open). "[11]

"***Proposition(1.3.50) :***

Every soft g-closed set is soft Ig-closed. "[11]

"***Corollary (1.3.51) :***

Every soft g-open set is soft Ig-open. "[8]

"***Remark(1.3.52):***

The converse of the proposition(1.3.50) is not in general true. "[2]

"***Proposition(1.3.53) :***

A soft set *(A,E)* is soft Ig-closed in a soft topological space (X,τ,E,I) if and only if *(F,E)*$\tilde{\subseteq}$*cl(A,E)-(A,E)* and *(F,E)* is soft closed implies *(F,E)*$\in$ *I*. "[11]

"***Proposition(1.3.54) :***

If *(F,E)* and *(G,E)* are soft Ig-closed sets in a soft topological space (X,τ,E,I), then *(F,E)*$\tilde{\cup}$*(G,E)* is also soft Ig-closed setin (X,τ,E,I). "[11]

"***Corollary(1.3.55) :***

If *(A,E)* and *(B,E)* are soft Ig-open sets in a soft topological space (X,τ,E,I), then *(A,E)*$\tilde{\cap}$*(B,E)* is soft Ig-open set in (X,τ,E,I). "[11]

Remark(1.3.56) :

We can show that the union of an infinite collection of soft Ig-closed sets is not soft Ig-closed set.

Example :

Let X= {1,2,3,….} , E = $\{e_1, e_2\}$, *I*={ ϕ_E } and τ = $\{X_E, \phi_E\}\tilde{\cup}$ {*(G$_n$,E)* ; n=1,2,3,… }, where, *(G$_n$,E)* be a soft set such that *(G$_n$,E)* ={(e_1,{n,n+1,n+2,… }), (e_2, ϕ) }. Let *(H$_m$,E)* be a soft set such that *(H$_m$,E)* ={(e_1,{1,2,3,4,…,m}) , (e_2, ϕ) }, $m \geq 10$. For each soft open set *(B,E)* such that *(H$_m$,E)*$\tilde{\subseteq}$*(B,E)*, $m \geq 10$. Then *cl((H$_m$,E))*= {(e_1,{1,2,3,4,…,m}), (e_2, ϕ) } , $m \geq 10$. Therefore, *cl(((H$_m$,E))- (B,E)*$=\phi_E \in I$, $m \geq 10$. Thus *(H$_m$,E)* is a soft Ig-closed set for each $m \geq 10$.

On the other hand $\tilde{\cup}_{m\geq 10}(H_m, E)$={(e_1,{1,2,3,4,… }), (e_2, ϕ) }= *(G$_1$,E)*. Then *(G$_1$,E)* $\tilde{\subseteq}$*(G$_1$,E)* and *(G$_1$,E)* is soft open set. Then, *cl(G$_1$,E)*= X_E. Therefore, *cl((G$_1$,E))- (G$_1$,E)*= {(e_1, ϕ) , (e_2,X) } $\notin$ *I. Thus,* $\tilde{\cup}_{m\geq 10}(H_m, E)$ is not soft Ig-closed set.

"***Theorem(1.3.57) :***

If *(F,E)* is soft Ig-closed in a soft topological space (X,τ,E,I) and *(F,E)* $\tilde{\subseteq}$ *(G,E)* $\tilde{\subseteq}$ *cl(F,E),* then *(G,E)* is soft Ig-closed in (X,τ,E,I)."[11]

"***Theorem(1.3.58):***

A soft set *(A,E)* is soft Ig-open in a soft topological space (X,τ,E,I) if and only if *(F,E)-(B,E)*$\tilde{\subseteq}$*int(A,E)* for some *(B,E)∈I,* whenever *(F,E)*$\tilde{\subseteq}$*(A,E)* and *(F,E)* is soft closed set in (X,τ,E,I). "[11]

"***Remark (1.3.59):***

The intersection of two soft Ig-closed sets need not be a soft Ig-closed. "[2]

"***Theorem(1.3.58):***

If *(A,E)* is soft Ig-closed and *(F,E)* is soft closed in a soft topological space (X,τ,E,I), then *(A,E)*$\tilde{\cap}$*(F,E)* is soft Ig-closed in (X,τ,E,I) . "[11]

"***Theorem(1.3.60) :***

Let $Y \subseteq X$ and *(F,E)*$\tilde{\subseteq} Y_E \tilde{\subseteq} X_E$. Suppose that *(F,E)* is soft Ig-closed in (X,τ,E,I). Then *(F,E)* is soft Ig-closed relative to the soft topological subspace Y_E of *X* and with respect to the soft ideal I_Y. "[11]

"***Theorem(1.3.61) :***

If *(A,E)* and *(B,E)* are soft separated and soft Ig-open sets in a soft topological space (X,τ,E,I), then *(A,E)*$\tilde{\cup}$*(B,E)* is soft Ig-open in (X,τ,E,I). "[11]

"***Corollary (1.3.62) :***

Let *(A,E)* and *(B,E)* be soft Ig-closed sets and suppose that $(A,E)^c$ and $(B,E)^c$ are soft separated in a soft topological space (X,τ,E,I). Then *(A,E)*$\tilde{\cap}$*(B,E)* is soft Ig-closed in (X,τ,E,I). "[11]

"***Theorem(1.3.63) :***

Let $M \tilde{\subseteq} X$ and *(A,E)*$\tilde{\subseteq} M_E \tilde{\subseteq} X_E$, *(A,E)* is soft Ig-open in (M,τ_M,E) and M_E is soft Ig-open in (X,τ,E,I). Then *(A,E)* is soft Ig-open in (X,τ,E,I). "[11]

"***Theorem(1.3.64):***

If $int(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(A,E)$ and *(A,E)* is soft Ig-open set in a soft topological space (X,τ,E,I), then *(B,E)* is soft Ig-open set in (X,τ,E,I)."[11]

CHAPTER TWO

SOFT STRONGLY GENERALIZED CLOSED SETS WITH RESPECT TO AN IDEAL IN SOFT TOPOLOGICAL SPACE

In this Chapter we make a mixture of the concepts which are given in chapter one to make the new concept soft strongly generalized closed set in soft topological space with respect to an ideal I (SSIg-closed). The complement of soft strongly generalized closed set in soft topological space with respect to an ideal *I* (SSIg-closed) is called soft strongly generalized open set in soft topological space with respect to an ideal *I* (SSIg-open) .

For any soft set (F,E) in a soft topological space with respect to an ideal I, we give in this Chapter the soft strongly generalized interior (closure, derived, border and boundary) set with *I* of (F,E).

2.1 SSIg-closed set .

In this section we define a soft strongly generalized closed sets with respect to an ideal in soft topological space and introduce some basic remarks, propositions and theorems about soft strongly generalized closed sets with respect to an ideal in soft topological space.

Definition(2.1.1) :

A soft set *(A,E)* in (X,τ,E,*I*) is said to be soft strongly generalized closed set with respect to an ideal *I*,(briefly SSIg- closed), if *cl(int(A,E))-(B,E)*$\in$ *I* whenever *(A,E)*$\tilde{\subseteq}$*(B,E)* and *(B,E)* is soft open set, the relative complement$(A,E)^c$ is soft strongly generalized open set with respect to an ideal ***I***,(briefly SSIg-open).

Example(2.1.2);

Let X={a,b,c} be the set of three cars under consideration and E={e_1(costly),e_2(Luxurious)}. Let *(A,E), (B,E), (C,E)* be three soft sets representing the attractiveness of the car which Mr. X, Mr. Y and M. Z are going to buy, τ = {ϕ_E , X_E, *(A,E)* , *(B,E)*,*(C,E)* } where *(A,E)* = {(e_1,{b}) ,

$(e_2,\{a\})\}$, *(B,E)*$=\{(e_1,\{b,c\}),(e_2,\{a,b\})\}$ and *(C,E)* $=\{(e_1,\{a,b\}),(e_2,\{a,c\})\}$. Then $\tau^c=\{\phi_E, X_E, (A,E)^c, (B,E)^c\}$ where $(A,E)^c=\{(e_1,\{a,c\}),(e_2,\{b,c\})\}$ and $(B,E)^c=\{(e_1,\{a\}),(e_2,\{c\})\}$, $(C,E)^c=\{(e_1,\{c\}),(e_2,\{b\})\}$. Let $I=\{\phi_E, (M,E), (H,E), (L,E)\}$ where $(M,E)=\{(e_1,\{a\}),(e_2,\phi)\}$ and $(H,E)=\{(e_1,\phi),(e_2,\{c\})\}$ and $(L,E)=\{(e_1,\{a\}),(e_2,\{c\})\}$.

Now $(B,E)\tilde{\subseteq}(B,E)$ and *(B,E)* is soft open set. Then *int(B,E)=(B,E)* and $cl(B,E)=X_E$. Therefore, $cl(int(B,E))-(B,E)=X_E-(B,E)=(B,E)^c=(L,E)\in I$. Hence, $cl(int(B,E))-(B,E)\in I$. Thus *(B,E)* is an SSIg-closed set.

On the other hand $(A,E)\tilde{\subseteq}(A,E)$ and *(A,E)* is soft open set . Then *int(A,E)* =*(A,E)* and $cl(A,E)=X_E$. Therefore, $cl(int(A,E))-(A,E)=X_E-(A,E)=(A,E)^c\notin I$. Hence *(A,E)* is not SSIg-closed set.

Proposition(2.1.3) :

In (X,τ,E,*I*) every soft closed set is an SSIg-closed set.

Proof :

Let *(A,E)* be a soft closed set in (X,τ,E,*I*). Let *(B,E)* be any soft open set in (X,τ,E,*I*) such that $(A,E)\tilde{\subseteq}(B,E)$. By definition of interior then $int(A,E)\tilde{\subseteq}(A,E)$, also By definition of closure and since *(A,E)* is soft closed set then $cl(int(A,E))\tilde{\subseteq}cl(A,E)=(A,E)\tilde{\subseteq}(B,E)$. Hence $cl(int(A,E))-(B,E)\tilde{\subseteq}(A,E)-(B,E)=\phi_E\in I$. *Thus (A,E)* is SSIg-closed set. □

Corollary(2.1.4) :

In (X,τ,E,*I*) every soft open set is an SSIg-open set.

Proof :

It is clear by Proposition(2.1.3). □

Corollary(2.1.5) :

Every soft subset of a soft discrete topological space with respect to an ideal *I* is an SSIg-closed set .

Proof :

Since every soft set in a soft discrete topological space is soft closed set, so it is an SSIg-closed set by Proposition(2.1.3).□

Remark(2.1.6):

In (X,τ,E,*I*), X_E and ϕ_E are SSIg-closed set.

Proof:

It is clear by Proposition(2.1.3) .□

Proposition(2.1.7) :

Every soft g-closed set is an SSIg-closed set.

Proof :

Let *(A,E)* be soft g- closed set. Suppose that *(A,E)*$\tilde{\subseteq}$*(B,E)* and *(B,E)* is soft open set . Since *(A,E)* is a soft g- closed set by hypothesis , then *cl(A,E)* $\tilde{\subseteq}$*(B,E)*. Since *int(A,E)*$\tilde{\subseteq}$*(A,E)* then *cl(int(A,E))*$\tilde{\subseteq}$*cl(A,E)*$\tilde{\subseteq}$*(B,E)*, therefore *cl(int(A,E))- (B,E)* = ϕ_E ∈*I*, hence *cl(int(A,E))- (B,E)*∈*I* whenever *(A,E)* $\tilde{\subseteq}$ *(B,E)* and *(B,E)* is soft open set. Hence, *(A,E)* is an SSIg-closed set.□

Remark(2.1.8) :

The converse of the Proposition(2.1.7) need not to be true.

Example :

Let X= {a,b,c} and E = {e_1, e_2} and τ = {ϕ_E, X_E,*(L,E)*,(B,E) } where *(L,E)* ={(e_1,{a}),(e_2,X)} and *(B,E)* = {(e_1,{a,b}) , (e_2,X) }. Then τ^c = {ϕ_E, X_E, $(L,E)^c$, $(B,E)^c$ } where $(L,E)^c$ ={(e_1,{b,c}) , (e_2,ϕ) } and $(B,E)^c$ = {(e_1,{c}) , (e_2,ϕ) } .

Let *I* = {ϕ_E , *(C,E)* , *(H,E)* , *(D,E)* } where *(C,E)*={(e_1,{b}) , (e_2,ϕ) } and (H,E)={(e_1,{b,c}) , (e_2,ϕ) } and (D,E)={(e_1,{c}) , (e_2,ϕ) } .

Now *(L,E)*$\tilde{\subseteq}$*(L,E)* and *(L,E)* is soft open set .

Then, *(L,E)* = *int(A,E)* and *cl(L,E)* = X_E .

Therefore, *cl(int(L,E))-(A,E)=* X_E *-(L,E)=(L,E)*^{c}*=(H,E)∈I*. Hence, *cl(int(L,E))-(L,E)∈I*. Thus, *(L,E)* is an SSIg-closed set. But *cl(L,E) =* $X_E \tilde{\not\subseteq}$ *(L,E)* for *(L,E)* $\tilde{\subseteq}$ *(L,E)* and *(L,E)* is soft open set. Therefore *(L,E)* is not soft g-closed set .□

Corollary(2.1.9) :

Every soft g-open set is an SSIg-open set.

Proof :

It is clear by Proposition(2.1.7). □

Theorem(2.1.10) :

Every soft Ig- closed set is an SSIg-closed set.

Proof :

Let *(A,E)* be soft Ig- closed set. Suppose that *(A,E)* $\tilde{\subseteq}$ *(B,E)* and *(B,E)* is soft open set. Since *(A,E)* is a soft Ig- closed set by hypothesis. Then *cl(A,E)-(B,E)∈I*. Since *int(A,E)* $\tilde{\subseteq}$ *(A,E)*. Then, *cl(int(A,E))*$\tilde{\subseteq}$*cl(A,E)*. Therefore *cl(int(A,E))- (B,E)* $\tilde{\subseteq}$ *cl(A,E)-(B,E)∈I*. By definition of an ideal we get *cl(int(A,E))- (B,E)∈I*. Thus, *(A,E)* is an SSIg-closed set.□

Remark(2.1.11) :

The converse of the Theorem(2.1.10) need not to be true.

Example :

Let X= {a,b,c}, E = {e_1, e_2} and τ ={ϕ_E, X_E,*(A,E),(B,E)*}, where *(A,E)* and *(B,E)* be a soft sets such that *(A,E)*={(e_1,{a}),(e_2,*X*) }, *(B,E)*= {(e_1,{a,c}),(e_2,X)}. Then τ^c = {ϕ_E, X_E, *(A,E)*^{c} , *(B,E)*^{c} } where *(A,E)*^{c} = {(e_1,{b,c}) , (e_2,ϕ) } and *(B,E)*^{c} = {(e_1,{b}) , (e_2,ϕ) } .

Let *I* = {ϕ , (C,E) , (H,E) , (D,E) } where *(C,E)*={(e_1,{b}),(e_2,ϕ)}, (H,E)={(e_1,{a,b}),(e_2,ϕ)}, (D,E)={(e_1,{a}) , (e_2,ϕ)}. Now *(D,E)*∈ *SS(X,E)* and *(D,E)*$\tilde{\subseteq}$*(A,E)* and *(A,E)*∈ τ. Then, *int(D,E)* = ϕ_E. So, *cl(int(D,E))* = ϕ_E. Therefore, *(D,E)* is an SSIg-closed set. But *cl(D,E)* = X_E, then *cl(D,E)- (A,E)* = X_E *-(A,E)* = *(A,E)*^{c} ∉ *I*. Therefore *(D,E)* is not soft Ig-closed set . □

Corollary(2.1.12) :

Every soft Ig-open set is an SSIg-open set.

Proof :

It is clear by Theorem(2.1.10) . □

Theorem(2.1.13) :

A soft set *(A,E)* in (X,τ,E,*I*) is a SSIg-closed set if and only if $(G,E)\tilde{\subseteq}$ *cl(int(A,E))-(A,E)* and *(G,E)* is soft closed set implies $(G,E)\in I$.

Proof :

Let *(A,E)* be SSIg- closed set. Assume that $(G,E)\ \tilde{\subseteq} cl(int(A,E))-(A,E)$ and *(G,E)* is soft closed set. Then $(G,E)\ \tilde{\subseteq}\ X_E$ *-(A,E),* so $(A,E)\ \tilde{\subseteq}\ X_E$ *-(G,E).* Therefore, X_E *-(G,E)* is soft open set and $(A,E)\ \tilde{\subseteq}\ X_E$ *-(G,E).* But *(A,E)* is SSIg-closed set, then $cl(int(A,E))–(X_E\text{-}(G,E))\in I$. But $(G,E)\tilde{\subseteq} cl(int(A,E))$ $-(A,E)\tilde{\subseteq} cl(int(A,E))\text{-}(X_E\text{-}(G,E))\in I$. By definition of an ideal *I* we get $(G,E)\in I$.

Conversely ,

Assume that $(G,E)\tilde{\subseteq} cl(int(A,E))-(A,E)$ and *(G,E)* is soft closed set implies $(G,E)\in I$. We need to prove that *(A,E)* is a SSIg-closed set .

Suppose that $(A,E)\tilde{\subseteq}(G,E)$ and $(G,E)\in\tau$. Then *cl(int(A,E))-(G,E)=* $cl(int(A,E))\tilde{\cap}\ X_E\text{-}(G,E)$. Since $(G,E)\ \in\tau$, then X_E *-(G,E)* is soft closed set, so for this $cl(int(A,E))\tilde{\cap}(X_E\text{-}(G,E)$ is soft closed set which is contained in *cl(int(A,E))-(G,E)*. By hypothesis $cl(int(A,E))-(G,E)\in I$. Therefore *(A,E)* is a SSIg-closed set . □

Theorem(2.1.14) :

If a soft subset *(A,E)* of (X,τ,E) is a SSIg-closed and if *cl(int(A,E))-(A,E)* contains a soft closed set *(G,E)* , then $cl(int(A,E))\tilde{\cap}(G,E)\in I$.

Proof :

Let *(A,E)* be SSIg- closed set. Assume that *(G,E)* is soft closed set such that $(G,E)\ \tilde{\subseteq} cl(int(A,E))-(A,E)$. Then $(G,E)\tilde{\subseteq}\ X_E$ *-(A,E),* so $(A,E)\tilde{\subseteq}\ X_E$ -

(G,E). Therefore, X_E -*(G,E)* is soft open set and *(A,E)*$\tilde{\subseteq}$ X_E -*(G,E)*. But *(A,E)* is SSIg-closed set. Then *cl(int(A,E))*–(X_E -*(G,E))* ∈*I*. Therefore , *cl(int(A,E))* $\tilde{\cap}$*(G,E)*∈*I* . □

Remark(2.1.15) :

The converse of the Theorem(2.1.14) need not to be true by the following example .

Example :

Let X= {a,b,c}, E = $\{e_1, e_2\}$and τ ={ϕ_E, X_E ,*(L,E),(B,E)*} where *(L,E)* ={(e_1,{a}),(e_2,{a,b}) }, *(B,E)*={(e_1,{a,c}),(e_2,X)}. Let *I*={ϕ , *(G,E)*} where *(G,E)*={(e_1,{b}),(e_2,ϕ) }, then τ^c = {ϕ_E, X_E, *(L,E)*^{c} , *(B,E)*^{c} }. Now *int(L,E)* = *(L,E)*, so *cl(int(L,E))*= X_E and since *(L,E)* is soft open set and *(L,E)*$\tilde{\subseteq}$*(L,E)*, but *cl(int(L,E))* - *(L,E)*= *(L,E)*^{c}∉ *I*, therefore *(L,E)* is not SSIg-closed. But on the other hand we have *(G,E)*= *(B,E)*^{c}, which it is soft closed set such that *(G,E)*$\tilde{\subseteq}$*cl(int(L,E))*- *(L,E)* and *cl(int(L,E))))*$\tilde{\cap}$*(G,E)*= *(G,E)*∈ *I*. □

Theorem(2.1.16) :

Let *(A,E)* and*(G,E)* are any soft sets in (X,τ,E,*I*). If *(A,E)* is an SSIg-closed and *(A,E)*$\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$*cl(int(A,E))*, then *(G,E)* is SSIg-closed set.

Proof :

Assume that *(A,E)* is an SSIg-closed and *(A,E)*$\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$*cl(int(A,E))*. To show that *(G,E)* is SSIg-closed set. *Suppose that (G,E)*$\tilde{\subseteq}$ *(F,E)* such that *(F,E)* is soft open set, *but (A,E)*$\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$ *(F,E), then (A,E)*$\tilde{\subseteq}$ *(F,E)* since *(A,E)* is SSIg-closed set, then *cl(int(A,E))-(F,E)*∈*I*. Now *(G,E)*$\tilde{\subseteq}$*cl(int(A,E)),* then *cl(int(G,E))*$\tilde{\subseteq}$*cl(int(A,E))*. This implies that *cl(int(G,E))-(F,E)*$\tilde{\subseteq}$*cl(int(A,E))-(F,E)*∈*I*. Therefore, *cl(int(G,E))-(F,E)*∈*I*. thus *(G,E)* is SSIg-closed set. □

Remark(2.1.17) :

The converse of the Theorem(2.1.16) need not to be true.

Example :

Let X= {a,b,c} and E = {e_1, e_2} and τ ={ϕ_E, X_E}, let *(A,E)* and *(G,E)* be a soft sets such that *(G,E)*={(e_1,{a}),(e_2,*X*) }, *(A,E)*={(e_1,{a}),(e_2,{a,b})}. Let *I*= {ϕ_E} . Then both *(A,E)* and *(G,E)* are SSIg-closed sets , but *cl(int(A,E))*= ϕ_E , thus *(A,E)* $\tilde{\subseteq}$ *(G,E)* $\tilde{\not\subseteq}$ *cl(int(A,E))*. On other hand if we consider that τ = {ϕ_E, X_E ,*(G,E)* }, then *(G,E)* is SSIg-closed set and *(A,E)*$\tilde{\subseteq}$*(G,E)* $\tilde{\subseteq}$*cl(int(A,E))*,but *(G,E)* is not SSIg-closed set. □

Corollary(2.1.18):

Let *(G,E)* and*(F,E)* are any soft sets in (X,τ,E,*I*). If *cl(int(F,E))* $\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$ *(F,E)* and if *(F,E)* is SSIg-open set, then *(G,E)* is SSIg-open set .

Proof :

Suppose that *cl(int(F,E))*$\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$*(F,E)* and *(F,E)* is SSIg-open set. We need to show that *(G,E)* is SSIg-open set. Then, X_E *-(F,E)*$\tilde{\subseteq}$ X_E *-(G,E)* $\tilde{\subseteq}$*cl(int(*X_E *-(F,E)))* and X_E *-(F,E)* is SSIg-closed set . By Theorem (2.1.16) we get X_E *-(G,E)* is SSIg-closed set .Therefore, *(G,E)* is SSIg-open set . □

Theorem(2.1.19) :

Let *(A,E)* and*(G,E)* are any soft sets in (X,τ,E,*I*). Then *(A,E)* is an SSIg-open set if and only if *(G,E)-(U,E)*$\tilde{\subseteq}$*int(A,E)* for some *(U,E)∈I* whenever *(G,E)*$\tilde{\subseteq}$ *(A,E)* and *(G,E)* is soft closed set .

Proof :

Suppose that *(A,E)* be an SSIg-open set and *(G,E)* is soft closed set such that *(G,E)* $\tilde{\subseteq}$ *(A,E)*. We need to show that *(G,E)-(U,E)* $\tilde{\subseteq}$*int(A,E)* for some (*U,E) ∈I*. Since *(G,E)* $\tilde{\subseteq}$ *(A,E)*. Then, X_E *-(A,E)* $\tilde{\subseteq}$ X_E *-(G,E)*. Since X_E *-(A,E)* is SSIg-closed set and *(G,E)* is soft open set. Then, *cl(int(*X_E *-(A,E)))* $\tilde{\subseteq}$ *(*X_E *-(G,E))*$\tilde{\cup}$*(U,E)* for some *(U,E)*$\in I$. Then X_E *-((*X_E *-(G,E))*$\tilde{\cup}$*(U,E))*$\tilde{\subseteq}$ X_E *-cl(int(*X_E *-(A,E)))*. Hence, *(G,E)-(U,E)*$\tilde{\subseteq}$*int(A,E)* for some *(U,E)∈I*.

Conversely,

Assume that $(G,E)\tilde{\subseteq}(A,E)$ and (G,E) is soft closed set implies that $(G,E)-(U,E)\tilde{\subseteq}int(A,E)$ for some $(U,E)\in I$.

We need to show that (A,E) is an SSIg-open, that is to say $X_E-(A,E)$ is an SSIg-closed set . Consider a soft open set (V,E) such that $X_E-(A,E)\tilde{\subseteq}(V,E)$. Then, $X_E-(V,E)\tilde{\subseteq}(A,E)$. Therefore, $(X_E-(V,E))-(U,E)\tilde{\subseteq}int(A,E)$ = $X_E-cl(int(X_E-(A,E)))$ for some $(U,E)\in I$. This gives that, $X_E-((V,E)\tilde{\cup}(U,E))\tilde{\subseteq}X_E-cl(int(X_E-(A,E)))$. Then, $cl(int(X_E-(A,E))\tilde{\subseteq}(V,E)\tilde{\cup}(U,E)$ for some $(U,E)\in I$. Therefore, $cl(int(X_E-(A,E))-(V,E)\in I$. Hence, $X_E-(A,E)$ is an SSIg-closed set. Therefore, (A,E) is an SSIg-open set . □

<u>Theorem(2.1.20) :</u>

Let $Y\tilde{\subseteq}X$ and (A,E) be a soft set in (Y,τ_Y,E). If (A,E) is a SSIg-closed set in (X,τ,E,I). then (A,E) is an SSI_Yg-closed relative to the soft space (Y,τ_Y,E) with respect to an ideal I_Y .

<u>Proof :</u>

Suppose that (A,E) is a SSIg-closed set in (X,τ,E,I). Let $(A,E)\tilde{\subseteq}(B,E)$. Then $(B,E)=(U,E)\tilde{\cap}Y_E$, where (U,E) is soft open set in (X,τ,E). Since $(U,E)\tilde{\cap}Y_E\tilde{\subseteq}(U,E)$,then $(A,E)\tilde{\subseteq}(U,E)$. Since (A,E) is a SSIg-closed set in (X,τ,E), then $cl(int(A,E))-(U,E)\in I$.

Therefore, $cl(int(A,E))-(B,E)=(cl(int(A,E))-((U,E)\tilde{\cap}Y_E)=(cl(int(A,E))-(U,E))\tilde{\cap}Y_E\tilde{\subseteq}(cl(int(A,E))-(U,E))\in I$. By definition of an ideal we get $(cl(int(A,E)))-(B,E)\in I_Y$. Thus (A,E) is an SSI_Yg-closed relative to the soft space in (Y,τ_Y,E) with respect to the ideal I_Y .□

<u>Theorem(2.1.21) :</u>

Let $Y\tilde{\subseteq}X$, $I=\{\phi_E\}$ and (A,E) be a soft set in (Y,τ_Y,E,I_Y). If (A,E) is a SSI_Yg-closed set over SSIg-closed set Y_E. then (A,E) is an SSIg-closed relative to (X,τ,E,I).

<u>Proof :</u>

Let *(A,E)*$\tilde{\subseteq}$*(B,E)* where *(B,E)* is soft open set in (X,τ,E). Since *(A,E)* =*(A,E)*$\tilde{\cap}Y_E \tilde{\subseteq}$*(B,E)*$\tilde{\cap}Y_E$, then *cl(int(A,E))-((B,E)*$\tilde{\cap}Y_E$*)*$\in I_Y$. Since *(A,E)* is an SSI_Yg-closed over Y, then *cl(int(A,E))*$\tilde{\subseteq}$*((B,E)*$\tilde{\cap}Y_E$*)* and Y_E $\tilde{\subseteq}$*((B,E)*$\tilde{\cup}$ *(cl(int(A,E)))*^{c}*)*. Since Y_E is an SSIg-closed, then *cl(int*Y_E*)-((B,E)*$\tilde{\cup}$ *(cl(int(A,E)))*^{c}*)* $\in I$. Then *cl(int(A,E)*$\tilde{\subseteq}$*cl(int*Y_E*)* $\tilde{\subseteq}$ *((B,E)*$\tilde{\cup}$ *(cl(int(A,E)))*^{c}*)* Therefore, *cl(int(A,E))-(B,E)*$\in$*I*. Thus *(A,E)* is an SSIg-closed.□

<u>Theorem (2.1.22):</u>

If *(A,E)* and *(G,E)* are two soft subsets of (X,τ,E,*I*) which are an SSIg-closed, then *(A,E)*$\tilde{\cup}$*(G,E)* is a SSIg-closed set .

<u>Proof :</u>

Suppose that *(A,E)* and *(G,E)* are two SSIg-closed sets .

We need to show that *(A,E)*$\tilde{\cup}$*(G,E)* is a SSIg-closed set .

Let *(U,E)* be a soft open set such that *(A,E)*∪*(G,E)* $\tilde{\subseteq}$ *(U,E)* . Then *(A,E)* $\tilde{\subseteq}$ *(U,E)* and *(G,E)* $\tilde{\subseteq}$ *(U,E)*. But both of them are SSIg-closed set, so *cl(int(A,E))-(U,E)*$\in$*I* and *cl(int(G,E))-(U,E)*$\in$*I*. Then *cl(int((A,E)*$\tilde{\cup}$*(G,E)))* *-(U,E)*$\tilde{\subseteq}$*cl(int(A,E)*$\tilde{\cup}$*int(G,E))-(U,E)* = *cl(int(A,E))*$\tilde{\cup}$*cl(int(G,E))-(U,E)* = *cl(int(A,E))-(U,E)*$\tilde{\cup}$*cl(int(G,E))-(U,E).* But *cl(int(A,E))-(U,E)*$\in$*I and cl(int(G,E))-(U,E)* $\in$*I*. By definition of an ideal we get, *cl(int(A,E))-(U,E)* $\tilde{\cup}$*cl(int(G,E))-(U,E)*$\in$*I*. Then *cl(int((A,E)*$\tilde{\cup}$*(G,E)))-(U,E)*$\in$*I* . Therefore, *(A,E)*$\tilde{\cup}$*(G,E)* is a SSIg-closed set. □

<u>Remark(2.1.23) :</u>

The infinite union of SSIg-closed sets need not to be SSIg-closed set in general.

<u>Example :</u>

Let X= {1,2,3,….} , E = $\{e_1, e_2\}$, *I*=$\{\phi_E\}$and τ = $\{X_E, \phi_E\}\tilde{\cup}\{$*(*$G_n$*,E)* ; n=1,2,3,… }, where, *(*G_n*,E)* is a soft set such that *(*G_n*,E)* ={(e_1,{n,n+1,n+2,… }) ,(e_2,ϕ) }. Let *(*H_m*,E)* be a soft set such that *(*H_m*,E)* ={(e_1,{2,3,4,…,m}) , (e_2,ϕ) }, $m \geq 10$. For each soft open set *(B,E)* such that *(*H_m*,E)*$\subseteq$ *(B,E),*

$m \geq 10$. Then *int(H$_m$,E)* = ϕ_E, $m \geq 10$. Hence *cl(int(H$_m$,E))* $= \phi_E$, $m \geq 10$. Therefore, *cl(int((H$_m$,E))-(B,E)*$= \phi_E$ *∈I*, $m \geq 10$. *Thus (H$_m$,E)* is a SSIg-closed set for each $m \geq 10$.

On the other hand $\tilde{\cup}_{m \geq 10}(H_m, E) = \{(e_1, \{2,3,4,\ldots\}), (e_2, \phi)\}$= *(G$_2$,E). Then (G$_2$,E)*$\tilde{\subseteq}$*(G$_2$,E)* and *(G$_2$,E)* is soft open set. Then, *int(G$_2$,E)=(G$_2$,E). Hence, cl(int(G$_2$,E))=cl(G$_2$,E)=* X_E. Therefore, *cl(int((G$_2$,E))-(G$_2$,E)=* $\{(e_1, \{1\})$, $(e_2$,X) $\} \notin$ *I. Thus,* $\tilde{\cup}_{m \geq 10}(H_m, E)$ is not SSIg-closed set.

Corollary(2.1.24) :

The intersection of two SSIg-open sets in (X,τ,E,*I*) is an SSIg-open.

Proof :

Suppose that *(A,E)* and *(G,E)* are two SSIg-open sets. We need to show that *(A,E)*$\tilde{\cap}$*(G,E)* is an SSIg-open set . Then X_E *-(A,E)* and X_E *-(A,E)* are two SSIg-closed sets . By Theorem(2.1.22) we get that X_E *-(A,E)*$\tilde{\cup}$ X_E *-(A,E)* = X_E *-((A,E)*$\tilde{\cap}$*(A,E))* is SSIg-closed set. Hence, *(A,E)*$\tilde{\cap}$*(G,E)* is a SSIg-open set. □

Remark(2.1.25) :

The intersection of two SSIg-closed sets need not to be SSIg-closed set in general.

Example :

Let X= {a,b,c} , E = $\{e_1, e_2\}$ and τ = { ϕ_E , X_E , *(A,E)* }, where *(A,E)* ,*(C,E)* and *(B,E)* are a soft sets such that *(A,E)*=$\{(e_1, \{b\})$, $(e_2, \phi)\}$, *(B,E)* =$\{(e_1, \{a,b\})$, $(e_2, \phi)\}$ and *(C,E)* = $\{(e_1, \{b,c\})$, $(e_2, \phi)\}$. Then $\tau^c = \{\phi_E, X_E$, *(A,E)*^{c} } where *(A,E)*^{c} =$\{(e_1, \{a,c\})$, $(e_2$,X) }. Let $I = \{\phi_E\}$.

(B,E) $\tilde{\subseteq}$ X_E and X_E is soft open set . Then, *int(B,E) = (A,E)* and *cl(int(B,E))* = *cl(A,E)*= X_E. Therefore, *cl(int((B,E))-* $X_E = X_E - X_E = \phi_E$ *∈I* . Hence, *(B,E)* is a SSIg-closed set , *(C,E)* $\tilde{\subseteq}$ X_E and X_E is soft open set. Then, *int(C,E) = (A,E)*

and *cl(int(C,E))* = *cl(A,E)*= X_E. Therefore, *cl(int((C,E))*- $X_E = X_E - X_E = \phi_E$ $\in I$. Hence, *(C,E)* is a SSIg-closed set.

Now, *(B,E)* $\tilde{\cap}$ *(C,E)* = {(e_1,{b}) , (e_2, ϕ) }=(*A,E*).

Since *(A,E)* $\tilde{\subseteq}$ *(A,E)* and *(A,E)* is soft open set. *int(A,E)* = *(A,E)* and *cl(int(A,E))* = X_E. Then, *cl(int(A,E))- (A,E)* = X_E *-(A,E)* = $(A,E)^c \notin I$ for *(A,E)* $\tilde{\subseteq}$ *(A,E)* and *(A,E)* is soft open set . Hence, *(B,E)*$\tilde{\cap}$*(C,E)* is not SSIg-closed set. Therefore The intersection of two SSIg-closed sets is not SSIg-closed set . □

Theorem(2.1.26) :

Let (A,E) and (G,E) be any soft sets in (X,τ,E,*I*). If *(A,E)* be a SSIg-closed set and *(G,E)* is a soft closed set. Then *(A,E)*$\tilde{\cap}$*(G,E)* is a SSIg-closed set.

Proof :

Let *(A,E)* be a SSIg-closed set and *(G,E)* is soft closed set .

We need to show that *(A,E)*$\tilde{\cap}$*(G,E)* is an SSIg-closed .

Let *(U,E)* be a soft open set such that *(A,E)*$\tilde{\cap}$*(G,E)* $\tilde{\subseteq}$ *(U,E)*.

Then, *(A,E)*$\tilde{\subseteq}$*(U,E)*$\tilde{\cup}$*(* X_E *-(G,E))* and *(U,E)*$\tilde{\cup}$*(* X_E *-(G,E))* is a soft open set.

But *(A,E)* is an SSIg-closed set. Then *cl(int(A,E))*- {*(U,E)*$\tilde{\cup}$*(* X_E *-(G,E))*} = {*cl(int(A,E))- (U,E)*}$\tilde{\cap}${*cl(int(A,E))-(* X_E *-(G,E))*} $\in I$. Therefore, *cl(int(A,E))* *-(* X_E *-(G,E))*$\in I$. Therefore, *cl(int((A,E)*$\tilde{\cap}$*(G,E)))*$\tilde{\subseteq}$ *cl(int((A,E)))*$\tilde{\cap}$*(G,E)*= *(cl(int((A,E)))*$\tilde{\cap}$*(G,E))-(* X_E *-(G,E))*. Hence, *cl(int((A,E)*$\tilde{\cap}$*(G,E)))-(U,E)* $\tilde{\subseteq}$*(cl(int((A,E)))*$\tilde{\cap}$*(G,E))-(U,E)*$\tilde{\cap}$*(* X_E *-(G,E))*$\tilde{\subseteq}$*cl(int((A,E))-((U,E)*$\tilde{\cup}$*(* X_E *-(G,E))* $\in I$. By definition of an ideal we get *cl(int((A,E)*$\tilde{\cap}$*(G,E)))-(U,E)*$\in I$. Thus, *(A,E)*$\tilde{\cap}$*(G,E)* is an SSIg-closed set .□

Proposition(2.1.27):

Let (X,τ_1,E_1) and (Y,τ_2,E_2) be two soft topological spaces with ideals I_1 and I_2 respectively. Then $I_1 \tilde{\times} I_2$ ={(V,E_1) $\tilde{\times}$ (U,E_2) ; $(V,E_1) \in I_1$,$(U,E_2) \in I_2$} is an ideal on the product soft topological space ($X \times Y$,$\tau_1 \times \tau_2$,$E_1 \times E_2$).

Proof:

Let $(V,E_1)\tilde{\times}(U,E_2)$, $(V_1,E_1)\tilde{\times}(U_1,E_2) \in I_1\tilde{\times}I_2$.

Then $(V,E_1)\tilde{\times}(U,E_2)\tilde{\cup}(V_1,E_1)\tilde{\times}(U_1,E_2)= (V,E_1)\tilde{\cup}(V_1,E_1)\tilde{\times}(U_1,E_2)\tilde{\cup}(U_1,E_2) \in I_1\tilde{\times}I_2$. If $(A,E_1)\tilde{\times}(B,E_2)\tilde{\subseteq}(V,E_1)\tilde{\times}(U,E_2)$, then $(A,E_1)\tilde{\times}(B,E_2) \in I_1\tilde{\times}I_2$. □

Proposition (2.1.28):

Let (X,τ_1,E_1) and (Y,τ_2,E_2) be two soft topological spaces with ideals I_1 and I_2 respectively. If (F,E_1) is an SSI_1g-closed set and (G,E_2) is an SSI_2g-closed set in (X,τ_1,E_1) and (Y,τ_2,E_2) respectively, then $(F,E_1) \tilde{\times} (G,E_2)$ is a SS$(I_1\tilde{\times}I_2)$g-closed set in $(X\times Y,\tau_1\times\tau_2,E_1\times E_2)$.

Proof:

Let $(V,E_1) \tilde{\times} (U,E_2)$ be a soft open set in $(X\times Y,\tau_1\times\tau_2,E_1\times E_2)$ such that $(F,E_1)\tilde{\times}(G,E_2)\tilde{\subseteq}(V,E_1)\tilde{\times}(U,E_2)$, Then $cl(int((F,E_1)\tilde{\times}(G,E_2)))-(V,E_1)\tilde{\times}(U,E_2)= cl(int((F,E_1)\tilde{\times}int((G,E_2)))-(V,E_1)\tilde{\times}(U,E_2)=cl(int((F,E_1))\tilde{\times}cl(int((G,E_2)))-(V,E_1)\tilde{\times}(U,E_2)= cl(int((F,E_1))) -(V,E_1)\tilde{\times}cl(int((G,E_2)))-(U,E_2) \in I_1\tilde{\times}I_2$. Hence $cl(int((F,E_1)\tilde{\times}(G,E_2)))-(V,E_1)\tilde{\times}(U,E_2) \in I_1\tilde{\times}I_2$. Thus, $(F,E_1)\tilde{\times}(G,E_2)$ is a SS$(I_1\tilde{\times}I_2)$g-closed set in $(X\times Y,\tau_1\times\tau_2,E_1\times E_2)$. □

Theorem(2.1.29) :

Let (F,E) and(G,E) are any soft sets in (X,τ,E,I). If (F,E) and (G,E) are separated soft SSIg-open sets, then $(F,E)\tilde{\cup}(G,E)$ is SSIg-open set .

Proof :

Suppose that (F,E) and (G,E) are separated soft SSIg-open sets and (U,E) be a soft closed subset of $(F,E)\tilde{\cup}(G,E)$.

Then , $(U,E)\tilde{\cap}cl(int(F,E))\tilde{\subseteq}(F,E)$ and $(U,E)\tilde{\cap}cl(int(G,E))\tilde{\subseteq}(G,E)$.

By hypothesis and Theorem(2.1.19) we get that ;

$\{(U,E)\tilde{\cap}cl(int(F,E))\}-(V,E)\tilde{\subseteq}int(F,E)$ and $\{(U,E)\tilde{\cap}cl(int(G,E))\}-(M,E) \tilde{\subseteq}int(G,E)$ for some (V,E) and (M,E) in I . This means that $\{(U,E) \tilde{\cap}cl(int(F,E))\}-int(F,E) \in I$ and $\{(U,E)\tilde{\cap}cl(int(G,E))\}-int(G,E) \in I$. Hence , $\{\{(U,E)\tilde{\cap}cl(int(F,E)\}-int(F,E)\}\tilde{\cup}cl(int(G,E)))-(int(F,E)\tilde{\cup}int(G,E)) \in I$. But

, $(U,E) = (U,E)\tilde{\cap}((F,E)\tilde{\cup}(G,E))\tilde{\subseteq}(U,E)\tilde{\cap}cl(int((F,E)\tilde{\cup}(G,E)))$ and we have, $(U,E)-(int((F,E)\tilde{\cup}(G,E)))\tilde{\subseteq}(U,E)\tilde{\cap}cl(int((F,E)\tilde{\cup}(G,E)))-int((F,E)\tilde{\cup}(G,E))$ $\tilde{\subseteq}$ $(U,E)\tilde{\cap}cl(int((F,E)\tilde{\cup}(G,E)))-int(F,E)\tilde{\cup}int(G,E) \in I$. Hence, $(U,E)-(H,E)$ $\tilde{\subseteq}int((F,E)\tilde{\cup}(G,E))$ for some $(H,E)\in I$. Therefore, $(F,E)\tilde{\cup}(G,E)$ is an SSIg-open set. □

Remark(2.1.30):

If the condition of separated is dropped then the Theorem(2.1.29) is not true in general.

Example :

Let X= {a,b,c} , E = $\{e_1,e_2\}$ and τ = $\{\phi_E$, X_E, (A,E) }, where (A,E) ,(C,E) and (B,E) are a soft sets such that (A,E) = $\{(e_1,\{b\}), (e_2,\phi)\}$, (B,E) = $\{(e_1,\{c\}), (e_2,X)\}$ and (C,E) = $\{(e_1,\{a\}), (e_2,X)\}$. Then $\tau^c = \{\phi_E, X_E,$ $(A,E)^c$ } where $(A,E)^c = \{(e_1,\{a,c\}), (e_2,X)\}$.

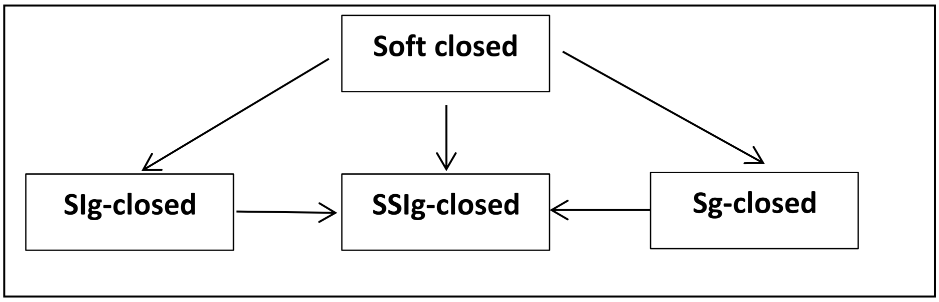
Let $I = \{\phi_E\}$. So $(B,E)^c = \{(e_1,\{a,b\}),(e_2,\phi)\}$ and $(C,E)^c = \{(e_1,\{b,c\}),(e_2,\phi)\}$, $(A,E)\tilde{\cap}cl(B,E)\neq\phi_E$ and $cl(A,E)\tilde{\cap}(B,E)\neq\phi_E$. So (A,E) and (B,E) are not separated. $(B,E)^c\tilde{\subseteq} X_E$ and X_E is soft open set. Then, $int(B,E)^c = (A,E)$ and $cl(int(B,E)^c) = cl(A,E)= X_E$. Therefore, $cl(int((B,E)^c)- X_E = X_E - X_E = \phi_E \in I$. Hence,$(B,E)^c$ is a SSIg-closed set. $(C,E)^c\tilde{\subseteq} X_E$ and X_E is soft open set . Then $int(C,E)^c = (A,E)$ and $cl(int(C,E)^c) = cl(A,E)= X_E$. Therefore, $cl(int((C,E)^c)- X_E = X_E - X_E = \phi_E \in I$. Hence, $(C,E)^c$ is a SSIg-closed set, so both $(C,E)$and (C,E) are SSIg-open sets.

Now, $(B,E)\cup(C,E)=\{(e_1,\{a,c\}),(e_2,X)\}$, so $\{(B,E)\cup(C,E)\}^c=\{(e_1,\{b\}),(e_2,\phi)\}$ = (A,E). Since $(A,E)\tilde{\subseteq}(A,E)$ and (A,E) is soft open set .

Then, $int(A,E) = (A,E)$ and $cl(int(A,E)) = X_E$. Therefore, $cl(int(A,E))- (A,E)$ = $X_E-(A,E) = (A,E)^c \notin I$ for $(A,E)\tilde{\subseteq}(A,E)$ and (A,E) is soft open set. Hence, $\{(B,E)\tilde{\cup}(C,E)\}^c$ is not SSIg-closed set. Therefore $(B,E)\tilde{\cup}(C,E)$ is not SSIg-open set . □

Note(2.1.31):

In the following diagram we discuss the relation between the kinds of soft closed sets



2.2 SSIg-Interior set

The aim of this section is to introduce the SSIg-interior and discuss some basic properties of the SSIg-interior sets in a soft topological space with an ideal *I*.

Definition(2.2.1) **:**

Let *(A,E)* be a soft set in (X,τ,E,*I*). Then the SSIg-interior of *(A,E)* is the union of all SSIg-open sets which are contained in *(A,E)* denoted by int^**(A,E)*.

Remark(2.2.2) :

X_E is an SSIg-neighborhood for each of its elements.

Theorem(2.2.3):

Let *(A,E)* be a soft set in (X,τ,E,*I*), then $int^*(A,E)\tilde{\subseteq}(A,E)$.

Proof:

Let $x\tilde{\in}int^*(A,E)$). Then there exists a SSIg-open set *(G,E)* such that $x\tilde{\in}$ $(G,E)\tilde{\subseteq}(A,E)$. Hence $x\tilde{\in}(A,E)$. Therefore $int^*(A,E)\tilde{\subseteq}(A,E)$. □

Remark(2.2.4) :

$int^*(A,E)\neq(A,E)$ in general.

Example :

Let X= {a,b} , E = $\{e_1,e_2\}$ and τ = { ϕ_E , X_E , *(A,E),(B,E), (C,E),(D,E)* }, where *(A,E) ,(B,E),(C,E)* and *(D,E)* be a soft sets such that *(A,E)* = {(e_1,{b}) , (e_2,ϕ) } , *(B,E)* = {(e_1,{b}) , (e_2,{a}) } , (C,E)= {(e_1,X) , (e_2,ϕ) } and *(D,E)* ={(e_1,X) ,(e_2,{a})} . Let I = { ϕ_E } .

Let *(S,E)*= {(e_1,{a}) , (e_2,*X*) } be a soft set. Then the soft set contained in *(S,E)* are {(e_1,{a}) , (e_2,*X*) },{(e_1,{a}) , (e_2,{a}) },{(e_1,{a}) , (e_2,ϕ) },{(e_1,ϕ), (e_2,{a}) }, {(e_1,{a}) , (e_2,{b}) },{(e_1,ϕ) , (e_2,{b}) },{(e_1,ϕ) , (e_2,X) } and ϕ_E . So we discusses each of these sets for SSIg-open.

$\{(e_1,\{a\}), (e_2,X)\}^c = \{(e_1,\{b\}), (e_2,\phi)\} \in \tau$, then $int\{(e_1,\{b\}), (e_2,\phi)\} = \{(e_1,\{b\}), (e_2,\phi)\}$ and $cl(int\{(e_1,\{b\}), (e_2,\phi)\}) = X_E$, therefore $cl(int\{(e_1,\{b\}), (e_2,\phi)\}) - \{(e_1,\{b\}), (e_2,\phi)\} \notin I$ .Thus $\{(e_1,\{b\}), (e_2,\phi)\}$ is not SSIg-closed set. For this $\{(e_1,\{a\}), (e_2,X)\}$ is not SSIg-open.

$\{(e_1,\{a\}), (e_2,\{b\})\}^c = \{(e_1,\{b\}), (e_2,\{a\})\} \in \tau$, then $int\{(e_1,\{b\}), (e_2,\{a\})\} = \{(e_1,\{b\}), (e_2,\{a\})\}$ and $cl(int\{(e_1,\{b\}), (e_2,\{a\})\}) = X_E$, therefore $cl(int\{(e_1,\{b\}), (e_2,\{a\})\}) - \{(e_1,\{b\}), (e_2,\{a\})\} \notin I$ .Thus $\{(e_1,\{b\}), (e_2,\{a\})\}$ is not SSIg-closed set. For this $\{(e_1,\{a\}), (e_2,\{b\})\}$ is not SSIg-open.

$\{(e_1,\phi), (e_2,\{b\})\}^c = \{(e_1,X), (e_2,\{a\})\}$. then $int\{(e_1,X), (e_2,\{a\})\} = \{(e_1,X), (e_2,\{a\})\}$, Hence $cl(int\{(e_1,X), (e_2,\{a\})\}) = X_E$. therefore $cl(int\{(e_1,\{b\}), (e_2,X)\}) - \{(e_1,X), (e_2,\{a\})\} = \{(e_1,\phi), (e_2,\{b\})\} \notin I$. Thus $\{(e_1,\{b\}), (e_2,X)\}$ is not SSIg-closed set. For this $\{(e_1,\{a\}), (e_2,\phi)\}$ is not SSIg-open.

$\{(e_1,\phi), (e_2,X)\}^c = \{(e_1,X), (e_2,\phi)\}$. Then $int\{(e_1,X), (e_2,\phi)\} = \{(e_1,X), (e_2,\phi)\}$, Hence $cl(int\{(e_1,X),(e_2,\phi)\}) = X_E$. Therefore $cl(int\{(e_1,X),(e_2,\phi)\}) - \{(e_1,X), (e_2,\phi)\} = \{(e_1,\phi), (e_2,X)\} \notin I$. Thus $\{(e_1,X), (e_2,\phi)\}$ is not SSIg-closed set. For this $\{(e_1,\phi), (e_2,X)\}$ is not SSIg-open.

$\{(e_1,\{a\}), (e_2,\{a\})\}^c = \{(e_1,\{b\}), (e_2,\{b\})\}$. then $int\{(e_1,\{b\}), (e_2,\{b\})\} = \{(e_1,\{b\}), (e_2,\phi)\}$, Hence $cl(int\{(e_1,\{b\}), (e_2,\{b\})\}) = X_E$. Thus $\{(e_1,\{b\}), (e_2,\{b\})\}$ is SSIg-closed set. For this $\{(e_1,\{a\}), (e_2,\{a\})\}$ is SSIg-open.

$\{(e_1,\phi), (e_2,\{a\})\}^c = \{(e_1,X), (e_2,\{b\})\}$. then $int\{(e_1,X), (e_2,\{b\})\} = \{(e_1,X), (e_2,\phi)\}$, Hence $cl(int\{(e_1,X), (e_2,\{b\})\}) = X_E$. therefore $cl(int\{(e_1,\{X\}), (e_2,\{b\})\}) - X_E = \phi_E \in I$. Thus $\{(e_1,\{X\}), (e_2,\{b\})\}$ is SSIg-closed set. For this $\{(e_1,\phi), (e_2,\{a\})\}$ is SSIg-open.

$\{(e_1,\{a\})\ ,\ (e_2,\phi)\ \}^c=\{(e_1,\{b\})\ ,\ (e_2,X)\ \}$. Then $int\{(e_1,\{b\})\ ,\ (e_2,X)\ \}=\{(e_1,\{b\}),(e_2,\{a\})\}$, Hence $cl(int\{(e_1,X)\ ,\ (e_2,\{b\})\}\ =\ X_E$. Therefore $cl(int\{(e_1,\{b\}),(e_2,X)\})$- $X_E=\phi_E \in I$. Thus $\{(e_1,\{b\}),(e_2,X)\ \}$ is SSIg-closed set. For this $\{(e_1,\{a\})\ ,(e_2,\phi)\ \}$ is SSIg-open.

Now, $int^*(A,E)=\cup\{(G,E);(G,E)$ is SSIg-open and $(G,E)\ \tilde{\subseteq}\ (A,E)\}=\{(e_1,\phi),(e_2,\{a\})\}\cup\{(e_1,\{a\}),(e_2,\phi)\ \}\cup\{(e_1,\{a\}),(e_2,\{a\})\}=\{(e_1,\{a\}),(e_2,\{a\})\}$, thus $(A,E)\not\subset int^*(A,E)$. □

Propositon (2.2.5):

Let (A,E) be any soft set in (X,τ,E,I). Then $int(A,E)\tilde{\subseteq} int^*(A,E)$.

Proof:

Let $x\tilde{\in} int(A,E)$. Then there exists a soft open set (V,E) such that $x\tilde{\in}$ $(V,E)\ \tilde{\subseteq}\ (A,E)$. But we have by Corollary(2.1.4) we get that (V,E) is SSIg-open set and $x\tilde{\in}\ (V,E)\tilde{\subseteq}(A,E)$. Hence $x\tilde{\in} int^*(A,E)$, thus $int(A,E)\tilde{\subseteq} int^*(A,E)$. □

Remark(2.2.6):

$int(A,E)\neq int^*(A,E)$ in general.

Example(2.2.7):

Let X= {a,b,c} , E = $\{e_1,e_2\}$ and τ = $\{\phi_E\ ,\ X_E\ \}$, let (A,E) be a soft set such that $(A,E)=\{(e_1,\{a\})\ ,\ (e_2,X)\ \}$. Let $I=\{\phi_E\}$. Then a$\tilde{\in}(A,E)\ \tilde{\subseteq}\ (A,E)$ and (A,E) is SSIg-open set , then a$\tilde{\in} int^*(A,E)$. But there is no soft open set (V,E) such that a$\tilde{\in}$ $(V,E)\ \tilde{\subseteq}\ (A,E)$, therefore a$\tilde{\notin} int(A,E)$. Thus $int^*(A,E)$ $\tilde{\not\subset} int(A,E)$. □

Theorem(2.2.8) :

If (A,E) is an SSIg-open set in (X,τ,E,I), then $int^*(A,E)=(A,E)$.

Proof:

For any soft subset (A,E) *of* (X,τ,E), $int^*(A,E)\tilde{\subseteq}(A,E)$. Let $x\tilde{\in}(A,E)$ *.Then* $x\tilde{\in}\ (A,E)\ \tilde{\subseteq}\ (A,E)$ and (A,E) is an SSIg-open which implies x is a

SSIg-interior of *(A,E)*. Hence $x \tilde{\in} int^*(A,E)$. Therefore $(A,E) \tilde{\subseteq} int^*(A,E)$. Hence $int^*(A,E) = (A,E)$. □

Corollary (2.2.9):

In (X,τ,E,I), $int^*(\phi_E) = \phi_E$ and $int^* X_E = X_E$.

Proof:

It is clear. □

Proposition(2.2.10):

Let *(A,E)* and *(B,E)* be soft sets in (X,τ,E,I). If *(B,E)* is any SSIg-open set contained in *(A,E)*, then $(B,E) \tilde{\subseteq} int^*(A,E)$.

Proof:

Let $x \tilde{\in} (B,E)$. Since *(B,E)* is SSIg-open set contained in *(A,E)*, x is a SSIg-interior point of *(A,E)*. $x \tilde{\in} int^*(A,E)$. Hence $(B,E) \tilde{\subseteq} int^*(A,E)$. □

Remark(2.2.11) :

Let *(A,E)* be a soft set in (X,τ,E,I), then $int^*(int^*(A,E)) = int^*(A,E)$.

Proof:

$int^*(int^*(A,E)) = \tilde{\cup}\{(U,E) ; (U,E) \in \text{SSIGO}(X) \text{ and } (U,E) \tilde{\subseteq} int^*(A,E)\} = \tilde{\cup}\{(U,E) ; (U,E) \in \text{SSIGO}(X) \text{ and } (U,E) \tilde{\subseteq} int^*(A,E) \tilde{\subseteq} (A,E)\} = \tilde{\cup}\{(U,E) ; (U,E) \in \text{SSIGO}(X) \text{ and } (U,E) \tilde{\subseteq} (A,E)\} = int^*(A,E)$. Therefore, $int^*(int^*(A,E)) = int^*(A,E)$. □

Proposition(2.2.12):

If *(A,E) and (B,E)* are any two soft sets in (X,τ,E,I) and $(A,E) \tilde{\cap} (B,E) = \phi_E$, then $int^*(A,E) \tilde{\cap} int^*(B,E) = \phi_E$.

Proof:

Given $(A,E) \tilde{\cap} (B,E) = \phi_E$. To prove that $int^*(A,E) \tilde{\cap} int^*(B,E) = \phi_E$. We have from Theorem (2.2.3) that $int^*(A,E) \tilde{\subseteq} (A,E)$ and $int^*(B,E) \tilde{\subseteq} (B,E)$. Therefore $int^*(A,E) \tilde{\cap} int^*(B,E) \tilde{\subseteq} (A,E) \tilde{\cap} (B,E) = \phi_E$, thus $int^*(A,E) \tilde{\cap} int^*(B,E) = \phi_E$. □

Theorem (2.2.13):

If *(A,E) and (B,E)* are any two soft sets in (X,τ,E,*I*), then $int^*(A,E) \tilde{\cup} int^*(B,E) \tilde{\subseteq} int^*\{(A,E) \tilde{\cup} (B,E)\}$.

Proof:

Since $(A,E) \tilde{\subseteq} (A,E) \tilde{\cup} (B,E)$ and $(B,E) \tilde{\subseteq} (A,E) \tilde{\cup} (B,E)$, then $int^*(A,E) \tilde{\subseteq} int^*\{(A,E) \tilde{\cup} (B,E)\}$ and $int^*(B,E) \tilde{\subseteq} int^*\{(A,E) \tilde{\cup} (B,E)\}$, therefore $int^*(A,E) \tilde{\cup} int^*(A,E) \tilde{\subseteq} int^*\{(A,E) \tilde{\cup} (B,E)\}$. □

Remark(2.2.14) :

$int^*(A,E) \tilde{\cup} int^*(A,E) \neq int^*((A,E) \tilde{\cup} (B,E))$ in general.

Example :

Let X= {a,b} , E = $\{e_1, e_2\}$, I={ϕ_E} and τ = {ϕ_E , X_E ,*(B,E)*}, let *(A,E)*and *(B,E)* are a soft sets such that *(A,E)* = {(e_1,{a}),(e_2,ϕ)} and *(B,E)* = {(e_1,{b}),(e_2,X)}. Then $int^*(A,E) = \phi_E$ since ϕ_E is the only SSIg-open which contained in *(A,E)*. Also $int^*(B,E) = (B,E)$ since *(B,E)* is SSIg-open set, therefore $int^*(A,E) \tilde{\cup} int^*(B,E)$= {($e_1$,{b}) , ($e_2$,X) }.

On the other hand $(A,E) \tilde{\cup} (B,E) = X_E$, so $int^*\{(A,E) \tilde{\cup} (B,E)\} = X_E$ by Corollary(2.2.9). Thus $int^*\{(A,E) \tilde{\cup} (B,E)\} \tilde{\not\subseteq} int^*(A,E) \tilde{\cup} int^*(B,E)$. □

Theorem (2.2.15):

If *(A,E) and (B,E)* are two soft sets in (X,τ,E,*I*), then $int^*((A,E) \tilde{\cap} (B,E)) = int^*(A,E) \tilde{\cap} int^*(B,E)$.

Proof:

Let $x \tilde{\in} int^*((A,E) \tilde{\cap} (B,E))$. Then there exists a SSIg-open set *(U,E)* such that $x \tilde{\in} (U,E) \tilde{\subseteq} (A,E) \tilde{\cap} (B,E) \tilde{\subseteq} (A,E)$. This implies that $x \tilde{\in} int^*(A,E)$. Also $x \tilde{\in} (U,E) \tilde{\subseteq} (A,E) \tilde{\cap} (B,E) \tilde{\subseteq} (B,E)$. So $x \tilde{\in} int^*(B,E)$. Hence $x \tilde{\in} int^*(A,E) \tilde{\cap} int^*(B,E)$, therefore $int^*((A,E) \tilde{\cap} (B,E)) \tilde{\subseteq} int^*(A,E) \tilde{\cap} int^*(B,E)$.

On the other hand let $x \tilde{\in} int^*(A,E) \tilde{\cap} int^*(B,E)\ ) \tilde{\subseteq} int^*(A,E)$ and $x \tilde{\in} int^*(A,E) \tilde{\cap} int^*(B,E)) \tilde{\subseteq} int^*(A,E)$, so $x \tilde{\in} int^*(A,E)$ and $x \tilde{\in} int^*(B,E)$ then there exist two SSIg-open sets *(U,E)*and*(V,E)* such that $x\tilde{\in}(U,E)\tilde{\subseteq}(A,E)$ *and* $x\tilde{\in}(V,E)\tilde{\subseteq}(B,E)$, then $x\tilde{\in}(U,E)\tilde{\cap}(V,E)\tilde{\subseteq}(A,E)$ and $x\tilde{\in}(U,E)\tilde{\cap}(V,E)\tilde{\subseteq}(B,E)$, so $x\tilde{\in}(U,E)\cap(V,E)\tilde{\subseteq}(A,E)\tilde{\cap}(B,E)$ and $(U,E)\cap(V,E)$ is an SSIg-open set by Corollary(2.1.24). Therefore $x\tilde{\in}int^*((A,E)\tilde{\cap}(B,E))$, hence $int^*(A,E)\tilde{\cap}int^*(B,E)\tilde{\subseteq}int^*((A,E)\tilde{\cap}(B,E))$. Thus $int^*((A,E)\tilde{\cap}(B,E))=int^*(A,E)\tilde{\cap}int^*(B,E)$. □

2.3 SSIg-closure set

Definition(2.3.1) :

For any soft subset *(A,E)* in a soft topological space (X,τ,E) with an ideal *I*, the SSIg-closure of *(A,E)*, denoted by $cl^*(A,E)$, is defined by the intersection of all SSIg-closed sets containing *(A,E)*.

Proposition(2.3.2) :

For a soft set *(A,E)* in (X,τ,E,I), then $(A,E)\tilde{\subseteq}cl^*(A,E)$.

Proof:

Let $x\tilde{\in}(A,E)$. By the definition of SSIg-closure of *(A,E)*, $x\tilde{\in}cl^*(A,E)$. So $(A,E)\tilde{\subseteq}cl^*(A,E)$. □

Remark (2.3.3):

If *(B,E)* is any SSIg-closed set and $(A,E)\tilde{\subseteq}(B,E)$ then $cl^*(A,E)\tilde{\subseteq}(B,E)$.

Proof:

By the definition of SSIg-closure, $cl^*(A,E)=\tilde{\cap}\{(F,E);(F,E)$ is SSIg-closed and $(A,E)\tilde{\subseteq}(F,E)\}$. Therefore $cl^*(A,E)$ is contained in every SSIg-closed set containing *(A,E)*. Since *(B,E)* is SSIg-closed set and $(A,E)\tilde{\subseteq}(B,E)$, $cl^*(A,E)\tilde{\subseteq}(B,E)$. □

Theorem (2.3.4):

If *(A,E)* is SSIg-closed set in (X,τ,E,I), then $(A,E)=cl^*(A,E)$.

Proof:

By the definition of SSIg-closure, $(A,E)\tilde{\subseteq} cl^*(A,E)$. Also $(A,E)\tilde{\subseteq}(A,E)$ and (A,E) is SSIg-closed set, and by Remark(2.3.3) , $cl^*(A,E)\tilde{\subseteq}(A,E)$. Hence $(A,E)= cl^*(A,E)$. □

Remark(2.3.5):

The following example shows that the converse of Theorem (2.3.4) need not be true in general.

Example :

Let X= {a,b,c} , E = { e_1, e_2 } , I={ ϕ_E } and τ = { ϕ_E , X_E , (A,E) }, where (A,E) be a soft set such that (A,E) = {(e_1 ,{a,b}) , (e_2 ,{b,c}) }.

Then (A,E) be a soft set we need to compute the SSIg-closure of it, so the soft sets which containing (A,E) are (A,E), (B,E) ={(e_1 ,{a,b}) , (e_2 ,X) }, (C,E) = {(e_1 ,X) , (e_2 ,{b,c}) }, X_E .

Now we shall check which of them is an SSIg-closed set.

For (A,E), $int(A,E)= (A,E)$ since it is soft open set, then $cl(int(A,E))= cl(A,E)= X_E$ .Since (A,E) is soft open and $(A,E)\tilde{\subseteq}(A,E)$, hence $cl(int(A,E))-(A,E)\notin$ I, thus (A,E) is not SSIg-closed set.

For (B,E), $int(B,E)= (A,E)$ since it is soft open set . then $cl(int(B,E))= cl(A,E)= X_E$ .Since X_E is the only soft open set for which $(B,E)\tilde{\subseteq} X_E$, hence $cl(int(A,E))- X_E \tilde{\in}$ I, thus (B,E) is SSIg-closed set.

For (C,E), $int(C,E)= (A,E)$ since it is soft open set . then $cl(int(C,E))= cl(A,E)= X_E$ .Since X_E is the only soft open set for which $(C,E)\tilde{\subseteq} X_E$, hence $cl(int(C,E))- X_E \in$ I, thus (C,E) is SSIg-closed set.

Therefore, the SSIg-closure of (A,E) is $cl^*(A,E) = (B,E)\tilde{\cap}(B,E)\tilde{\cap} X_E =(A,E)$, thus $cl^*(A,E) = (A,E)$, but (A,E) is not SSIg-closed set. □

Note(2.3.6):

The Example in Remark(2.3.5) shows that $cl^*(A,E)$ is not SSIg-closed set in general.

Corollary(2.3.7) :

In (X,τ,E,I), $cl^*(\phi_E)=\phi_E$ and $cl^*(X_E)=X_E$.

Proof:

Follows from Remark(2.1.6), Theorem (2.3.4). □

Theorem (2.3.8):

If *(A,E)* and *(B,E)* are any soft sets in (X,τ,E,I) and $(A,E)\tilde{\subseteq}(B,E)$ *then* $cl^*(A,E)\tilde{\subseteq}cl^*(B,E)$.

Proof:

Let $x\tilde{\notin}cl^*(B,E)$. By the definition of $cl^*(B,E)$, then there exsits SSIg-closed set *(F,E)* such that $(B,E)\tilde{\subseteq}(F,E)$ and $x\tilde{\notin}(F,E)$. Since $(A,E)\tilde{\subseteq}(B,E)$, $(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(F,E)$ and $x\tilde{\notin}(F,E)$ which is SSIg-closed, so $x\tilde{\notin}cl^*(A,E)$. That is $cl^*(A,E)\tilde{\subseteq}cl^*(B,E)$. □

Theorem(2.3.9) :

If *(A,E)* and *(B,E)* are any two soft sets in (X,τ,E,I), then $cl^*(A,E)\tilde{\cup}cl^*(B,E)\tilde{\subseteq}cl^*((A,E)\tilde{\cup}(B,E))$.

Proof:

Let *(A,E)* and *(B,E)* be any two soft sets in (X,τ,E). Clearly $(A,E)\tilde{\subseteq}(A,E)\tilde{\cup}(B,E)$ and $(B,E)\tilde{\subseteq}(A,E)\tilde{\cup}(B,E)$. By Theorem(2.3.8), we have $cl^*(A,E)\tilde{\subseteq}cl^*((A,E)\tilde{\cup}(B,E))$ and $cl^*(B,E)\tilde{\subseteq}cl^*((A,E)\tilde{\cup}(B,E))$. Hence $cl^*(A,E)\tilde{\cup}cl^*(B,E)\tilde{\subseteq}cl^*((A,E)\tilde{\cup}(B,E))$. □

Remark (2.3.10) :

$cl^*(A,E)\tilde{\cup}cl^*(B,E)\neq cl^*((A,E)\tilde{\cup}(B,E))$ in general.

Example :

Let X= {a,b,c} , E = $\{e_1,e_2\}$, I=$\{\phi_E\}$ and τ = $\{\phi_E, X_E$, *(A,E)* ,(B,E) ,(C,E) ,(D,E),(H,E) }, where *(A,E)*,*(B,E)*,*(C,E)*,*(D,E)* and *(H,E)* are soft sets such that *(A,E)* = {(e_1,{a,b }) , (e_2,{a}) },*(B,E)* ={(e_1,{a,b}) ,(e_2,{b})}, *(C,E)* = {(e_1,{a,b }) ,(e_2,X) }, *(D,E)* = {(e_1,{a,b}) , (e_2,{a,b}) } and *(H,E)* = {(e_1,{a,b }) , (e_2,ϕ) }.

Now *(A,E)* = {(e_1,{a,b }) , (e_2,{a})} and *(B,E)* = {(e_1,{a,b}) , (e_2,{b})} . Then the SSIg-closed set which contains *(A,E)* are X_E, {(e_1,{a,b}), (e_2,{a,c})}, {(e_1,X) , (e_2,{a,b})}, {(e_1,X) , (e_2,{a,c})} and {(e_1,X) , (e_2,{a})}. Therefore cl^**((A,E)*= $X_E \tilde{\cap}$ {(e_1,{a,b}),(e_2,{a,c})} $\tilde{\cap}$ {(e_1,X) , (e_2,{a,b})} $\tilde{\cap}$ {(e_1,X) , (e_2,{a,c})} $\tilde{\cap}$ {(e_1,X) , (e_2,{a})}={(e_1,{a,b}),(e_2,{a})}. And the SSIg-closed set which contains *(B,E)* are X_E,{(e_1,{a,b}), (e_2,{a,b})},{(e_1,X),(e_2,{a,b})},{(e_1,X), (e_2,{b,c})} and {(e_1,X) , (e_2,{b})}. Therefore cl^**((B,E)*= $X_E \tilde{\cap}$ {(e_1,{a,b}),(e_2,{a,b})} $\tilde{\cap}$ {(e_1,X),(e_2,{a,b})} $\tilde{\cap}$ {(e_1,X) , (e_2,{b,c})} $\tilde{\cap}$ {(e_1,X) , (e_2,{b})}={(e_1,{a,b}),(e_2,{b})}.

On the other hand *(A,E)* $\tilde{\cup}$ *(B,E)*= {(e_1,{a,b}),(e_2,{a,b})}, Then the SSIg-closed set which contains *(A,E)* $\tilde{\cup}$ *(B,E)* are X_E and {(e_1,X) , (e_2,{a,b})}. Therefore cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*}= $X_E \tilde{\cap}$ {(e_1,X),(e_2,{a,b})}={(e_1,X), (e_2,{a,b})}. Hence cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*}={(e_1,X) , (e_2,{a,b})} $\not\subset$ {(e_1,{a,b}),(e_2,{a,b})}= cl^**(A,E)* $\tilde{\cup}$ cl^**(B,E)*.Thus cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*} $\tilde{\not\subset}$ cl^**(A,E)* $\tilde{\cup}$ cl^**(B,E)*. □

Remark (2.3.11) :

If *(A,E)* and *(B,E)* are any soft sets in (X,τ,E,*I*) and cl^**(A,E)* $\tilde{\cap}$ cl^**(B,E)*= ϕ_E then *(A,E)* $\tilde{\cap}$ *(B,E)*= ϕ_E .

Proof:

Let cl^**(A,E)* $\tilde{\cap}$ cl^**(B,E)*= ϕ_E. To prove that *(A,E)* $\tilde{\cap}$ *(B,E)*= ϕ_E. Since *(A,E)* $\tilde{\cap}$ *(B,E)* $\tilde{\subseteq}$ cl^**(A,E)* $\tilde{\cap}$ cl^**(B,E)*= ϕ_E . Hence *(A,E)* $\tilde{\cap}$ *(B,E)*= ϕ_E . □

Theorem (2.3.12):

If *(A,E)* and *(B,E)* are any subsets of a soft topological space (X,τ,E) with an ideal *I, then* cl^*((*A,E)* $\tilde{\cap}$ *(B,E)*) $\tilde{\subseteq}$ cl^**(A,E)* $\tilde{\cap}$ cl^**(B,E)*.

Proof:

Let *(A,E)* and *(B,E)* be sets in (X,τ,E,*I*). Also $(A,E)\tilde{\cap}(B,E) \tilde{\subseteq} (A,E)$ & $(A,E)\tilde{\cap}(B,E) \tilde{\subseteq} (B,E)$. Therefore by Theorem(2.3.8), we have $cl^{*}((A,E)\tilde{\cap}(B,E)) \tilde{\subseteq} cl^{*}(A,E)$ *and* $cl^{*}((A,E)\tilde{\cap}(B,E)) \tilde{\subseteq} cl^{*}(B,E)$.Therefore $cl^{*}((A,E)\tilde{\cap}(B,E)) \tilde{\subseteq} cl^{*}(A,E)\tilde{\cap}\, cl^{*}(B,E)$. □

Remark(2.3.13) :

$$cl^{*}((A,E)\tilde{\cap}(B,E)) \neq cl^{*}(A,E)\tilde{\cap}\, cl^{*}(B,E).$$

Example :

Let X= {a,b,c} , E = $\{e_1, e_2\}$, I=$\{\phi_E\}$ and τ = $\{\phi_E, X_E$, *(A,E)* ,(B,E) ,(C,E) ,(D,E),(H,E) }, where *(A,E),(B,E),(C,E),(D,E)* and *(H,E)* are soft sets such that *(A,E)* = {(e_1,{a }) , (e_2,{a,b}) }, *(B,E)* = {(e_1,X) , (e_2,{a,c})}, *(C,E)* = {(e_1,{a }) , (e_2,X) }, *(D,E)* = {(e_1,{a}) , (e_2,{a}) } and *(H,E)* = {(e_1,{a }) , (e_2,{a,c}) }. We have *(A,E)* = {(e_1,{a }) , (e_2,{a,b})} and *(M,E)* = {(e_1,{b }) , (e_2,{b,c})}. Then $cl^{*}((A,E)\tilde{\cap}(M,E))=cl^{*}\{(e_1,\phi),(e_2,\{b\})\}=\{(e_1,\phi),(e_2,\{b\})\}$. Now we compute $cl^{*}((A,E))$. *(A,E)* is soft open set, it is not SSIg-closed set, the SSIg-closed sets containing *(A,E)* are {(e_1,X) , (e_2,{a,b})}, {(e_1,{a,b}) , (e_2,{a,b})} and {(e_1,{a,b}) , (e_2,X)} , then $cl^{*}((A,E))=$ {(e_1,X) , (e_2,{a,b})}$\tilde{\cap}${(e_1,{a,b}) , (e_2,{a,b})}$\tilde{\cap}${(e_1,{a,b}) , (e_2,X)}= {(e_1,{a,b}) , (e_2,{a,b})}. For $cl^{*}((M,E))$ since *(M,E)* is SSIg-closed set, then $cl^{*}((M,E))=(M,E)$, therefore $cl^{*}(A,E)\tilde{\cap}cl^{*}(M,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$. Thus $cl^{*}(A,E)\tilde{\cap}cl^{*}(M,E) \tilde{\not\subseteq} cl^{*}((A,E)\tilde{\cap}(M,E))$. □

Proposition(2.3.14) :

For an $x \in X$, $x \tilde{\in} cl^{*}(A,E)$ if and only if $(V,E)\tilde{\cap}(A,E) \neq \phi_E$ for every SSIg-open set *(V,E)* containing x.

Proof:

Let $x \in X, x \tilde{\in} cl^{*}(A,E)$. To prove $(V,E)\tilde{\cap}(A,E) \neq \phi_E$ for every SSIg-open set *(V,E)* containing x. We prove this by contradiction. Suppose that there exists a SSIg-open set *(V,E)* containing x such that $(V,E)\tilde{\cap}(A,E) = \phi_E$. Then

(A,E) $\tilde{\subseteq}$ *(V,E)*^{c} and *(V,E)*^{c} is SSIg-closed set . Hence cl^**(A,E)* $\tilde{\subseteq}$ *(V,E)*^{c}. Therefore cl^**(A,E)* $\tilde{\cap}$ *(V,E)*= ϕ_E . This implies that $x \tilde{\notin} cl^*$*(A,E)* which is a contradiction. So *(V,E)* $\tilde{\cap}$ *(A,E)*≠ ϕ_E for every SSIg-open set *(V,E)* containing x.

Conversely, let *(V,E)* $\tilde{\cap}$ *(A,E)*≠ ϕ_E for every SSIg-open set *(V,E)* containing x. To prove that $x \tilde{\in} cl^*$*(A,E)*. We prove this by contradiction. Assume that $x \tilde{\notin}$ cl^**(A,E)*.Then there exists a SSIg-closed set *(F,E)* such that *(A,E)* $\tilde{\subseteq}$ *(F,E)* and $x \tilde{\notin}$ *(F,E)*, then *(F,E)*^{c} is SSIg-open set containing x with *(F,E)*^{c} $\tilde{\cap}$ *(A,E)*=ϕ_E which is a contradiction. Hence $x \tilde{\in} cl^*$*(A,E)*. □

<u>Proposition(2.3.15):</u>

Let *(A,E)* be any soft set in (X,τ,E,*I*). Then cl^**(A,E)* $\tilde{\subseteq}$ *cl(A,E)*.

<u>Proof:</u>

let $x \tilde{\notin}$ *cl(A,E)* so by Proposition (2.3.14) there exists a soft open set *(V,E)* such that $x \tilde{\in}$ *(V,E)* and *(V,E)* $\tilde{\cap}$ *(A,E)*= ϕ_E. But by Corollary (2.1.4) *(V,E)* is SSIg-open set and $x \tilde{\in}$ *(V,E)* and *(V,E)* $\tilde{\cap}$ *(A,E)*= ϕ_E, therefore $x \tilde{\notin}$ cl^**(A,E)*. Thus cl^**(A,E)* $\tilde{\subseteq}$ *cl(A,E)*. □

<u>Remark(2.3.16):</u>

cl_g*(A,E)* $\tilde{\subseteq}$ cl^**(A,E)* in general and also cl^**(A,E)* ≠ *cl(A,E)*.

<u>Example :</u>

Let X= {a,b,c} , E = {e_1, e_2}, I={ϕ_E} and τ = {ϕ_E , X_E}, let *(A,E)* be a soft set such that *(A,E)* = {(e_1,{a,b }) , (e_2,{a}) }. Then the SSIg-closed set which containing *(A,E)* are X_E , {(e_1,{a,b}),(e_2,{a,c})}, {(e_1,X) , (e_2,{a,b})}, {(e_1,X) , (e_2,{a,c})} and {(e_1,X) , (e_2,{a})}. Therefore cl^**((A,E)*= X_E $\tilde{\cap}$ {(e_1,{a,b}),(e_2,{a,c})} $\tilde{\cap}$ {(e_1,X) , (e_2,{a,b})} $\tilde{\cap}$ {(e_1,X) ,(e_2,{a,c})} $\tilde{\cap}$ {(e_1,X) , (e_2,{a})}={(e_1,{a,b}),(e_2,{a})}.

On the other hand *cl((A,E)*= X_E . Thus *cl(A,E)* $\tilde{\not\subset}$ cl^**(A,E)*. □

Theorem(2.3.17) :

If *(A,E)* is a soft subset in (X,τ,E,I), then $cl^*(A,E) = cl^*\{\, cl^*(A,E)\}$.

Proof:

$cl^*\{\, cl^*(A,E)\} = \tilde{\cap}\{(U,E);\ (U,E)\tilde{\in}$ SSIGC(X) *and* $cl^*(A,E)\tilde{\subseteq}(U,E)\,\} = \tilde{\cap}\{(U,E);\ (U,E)\tilde{\in}$ SSIGC(X) *and* $(A,E)\tilde{\subseteq}cl^*(A,E)\tilde{\subseteq}(U,E)\} = \tilde{\cap}\{(U,E);\ (U,E)$ $\tilde{\in}$ SSIGC(X) *and* $(A,E)\tilde{\subseteq}(U,E)\}$. Thus, $cl^*(A,E) = cl^*\{\, cl^*(A,E)\}$. □

Theorem(2.3.18) :

For any soft set *(A,E)* in (X,τ,E,I), $\{cl^*(A,E)\}^c = int^*(A,E)^c$.

Proof:

For any point $x\in X, x\tilde{\in}\{cl^*(A,E)\}^c$ implies $x\tilde{\notin}cl^*(A,E)$. Then there exists SSIg-open set U containing x, $(A,E)\tilde{\cap}(U,E)=\phi_E$. So $x\tilde{\in}(U,E)\ \tilde{\subseteq}\ (A,E)^c$. Thus $x\tilde{\in}\ int^*(A,E)^c$. Conversely, let $x\tilde{\in}int^*(A,E)^c$. There exists a SSIg-open set (U,E) such that $x\tilde{\in}\ (U,E)\subset(A,E)^c$ that is $x\tilde{\in}\ (U,E)$ and $(U,E)\tilde{\cap}(A,E)$ and by Theorem (2.4.14). So $x\tilde{\notin}\ cl^*(A,E)$. This implies that $x\tilde{\in}\{\, cl^*(A,E)\}^c$. □

Remark (2.3.19):

For any soft set *(A,E)* in (X,τ,E,I), $\{int^*(A,E)\}^c = cl^*(A,E)^c$.

Proof:

Follows from Theorem (2.3.18). □

Remark (2.3.20):

For any soft set *(A,E)* in (X,τ,E,I), $cl^*(A,E) =\{\, int^*(A,E)^c\}^c$.

Proof:

It is clear by Theorem (2.3.18). □

Theorem(2.3.21) :

For any soft set *(A,E)* in (X,τ,E,I), $int^*(A,E)=\{\, cl^*(A,E)^c\}^c$.

Proof:

Let $x\tilde{\in}int^*(A,E)$. Then there exists a SSIg-open set *(U,E)* such that $x\tilde{\in}$ $(U,E)\ \tilde{\subseteq}\ (A,E)$. Hence $x\tilde{\notin}cl^*(A,E)^c$. Therefore $x\tilde{\in}\{cl^*(A,E)^c\ \}^c$. Hence $int^*(A,E)\ \tilde{\subseteq}\ \{cl^*(A,E)^c\}^c$. Conversely, let $x\tilde{\in}\{cl^*(A,E)^c\ \}^c$. This implies that x

$\tilde{\notin}$ $cl^*(A,E)^c$. Then there exists a SSIg-open set *(U,E)* with $x\tilde{\in}(U,E)\tilde{\cap}(A,E)^c$= ϕ_E . That is there exists a SSIg-open set *(U,E)* with $x\tilde{\in}(U,E)$ $\tilde{\subseteq}$ *(A,E)*. So $x\tilde{\in}$ $int^*(A,E)$. Therefore $\{cl^*(A,E)^c\}^c$ $\tilde{\subseteq}$ $int^*(A,E)$. Hence $int^*(A,E)=\{cl^*(A,E)^c\}^c$. □

2.4 SSIg-derived set

Definition(2.4.1) :

Let *(A,E)* be a soft set in (X,τ,E,I), $x\in X$ is called SSIg-limit point of *(A,E)* if for each SSIg-open set *(U,E)* containing x such that $(U,E)\tilde{\cap}(A,E)$-$\{x\}\neq\phi_E$.

Example :

Let X= {a,b,c} , E = $\{e_1,e_2\}$ and τ = $\{\phi_E$, X_E , *(A,E)*} where *(A,E)* be a soft set such that *(A,E)* = $\{(e_1,\{a\})$, $(e_2,X)\}$. Let $I=\{\phi_E\}$. Then a$\tilde{\in}$*(A,E)* and a$\tilde{\in}$*(B,E)* =$\{(e_1,\{a\})$, $(e_2,\{a\})\}$ and *(B,E)* is SSIg-open set since $(B,E)^c$= $\{(e_1,\{b,c\})$, $(e_2,\{b,c\})\}$. Then $int(B,E)^c$ = ϕ_E and $cl(int((B,E)^c))$= ϕ_E , therefore $cl(int((B,E)^c))$- $X_E=\phi_E\in I$. But $(B,E)\tilde{\cap}(A,E)$- $\{a\}=\phi_E$. Thus a is not SSIg-limit point of *(A,E)*. □

Note(2.4.3) :

The set of all SSIg-limit points of *(A,E)* denoted by $\dot{D}(A,E)$ (the SSIg-derived set of *(A,E)*) .

Proposition(2.4.4) :

For any two soft subsets *(A,E)* and *(B,E)* in (X,τ,E,I), if $(A,E)\tilde{\subseteq}(B,E)$, then $\dot{D}(A,E)\tilde{\subseteq}\dot{D}(B,E)$.

Proof:

Let $x \tilde{\notin} \dot{D}(B,E)$. Then there exists a SSIg-open set (U,E) such that $x \tilde{\in} (U,E)$ and $(U,E) \tilde{\cap} (B,E)-\{x\}=\phi_E$. But $(A,E) \tilde{\subseteq} (B,E)$, then $(U,E) \tilde{\cap} (A,E)-\{x\} \tilde{\subseteq} (U,E) \tilde{\cap} (B,E)-\{x\}=\phi_E$. Hence $(U,E) \tilde{\cap} (A,E)-\{x\}=\phi_E$, therefore $x \tilde{\notin} \dot{D}(A,E)$. Thus $\dot{D}(A,E) \tilde{\subseteq} \dot{D}(B,E)$. □

Proposition(2.4.5) :

For any two soft sets (A,E) and (B,E) in (X,τ,E,I), then $\dot{D}(A,E) \tilde{\cup} \dot{D}(B,E)= \dot{D}((A,E) \tilde{\cup} (B,E))$.

Proof:

Since $(A,E) \tilde{\subseteq} ((A,E) \tilde{\cup} (B,E))$ and $(B,E) \tilde{\subseteq} ((A,E) \tilde{\cup} (B,E))$. Then by Proposition(2.5.3) we get that $\dot{D}(A,E) \tilde{\subseteq} \dot{D}((A,E) \tilde{\cup} (B,E))$ and $\dot{D}(B,E) \tilde{\subseteq} \dot{D}((A,E) \tilde{\cup} (B,E))$. Therefore $\dot{D}(A,E) \tilde{\cup} \dot{D}(B,E) \tilde{\subseteq} \dot{D}((A,E) \tilde{\cup} (B,E))$.

On the other hand let $x \tilde{\notin} \dot{D}(A,E) \tilde{\cup} \dot{D}(B,E)$, then $x \tilde{\notin} \dot{D}(A,E)$ and $x \tilde{\notin} \dot{D}(B,E)$ that is there exists two SSIg-open sets $(V,E)$and(U,E) containing x such that $(U,E) \tilde{\cap} (A,E)-\{x\}=\phi_E$, and $(V,E) \tilde{\cap} (B,E)-\{x\}=\phi_E$, since $(U,E) \tilde{\cap} (V,E)$ is SSIg-open set by Theorem (2.1.22) and $((U,E) \tilde{\cap} (V,E)) \tilde{\cap} ((A,E) \tilde{\cup} (B,E)) -\{x\} = \{((U,E) \tilde{\cap} (V,E)) \tilde{\cap} (A,E) -\{x\}\} \tilde{\cup} \{((U,E) \tilde{\cap} (V,E)) \tilde{\cap} (B,E)-\{x\}\}= \phi_E$, therefore $x \tilde{\notin} \dot{D}((A,E) \tilde{\cup} (B,E))$, hence $\dot{D}((A,E) \tilde{\cup} (B,E)) \tilde{\subseteq} \dot{D}(A,E) \tilde{\cup} \dot{D}(B,E)$. Thus $\dot{D}(A,E) \tilde{\cup} \dot{D}(B,E)= \dot{D}((A,E) \tilde{\cup} (B,E))$. □

Proposition (2.4.6):

For any two soft sets (A,E) and (B,E) in (X,τ,E,I), then $\dot{D}((A,E) \tilde{\cap} (B,E)) \tilde{\subseteq} \dot{D}(A,E) \tilde{\cap} \dot{D}(B,E)$.

Proof:

Since $(A,E) \tilde{\cap} (B,E) \tilde{\subseteq} (A,E)$ and $(A,E) \tilde{\cap} (B,E) \tilde{\subseteq} (B,E)$, then $\dot{D}((A,E) \tilde{\cap} (B,E)) \tilde{\subseteq} \dot{D}(A,E)$ and $\dot{D}((A,E) \tilde{\cap} (B,E)) \tilde{\subseteq} \dot{D}(B,E)$. Therefore $\dot{D}((A,E) \tilde{\cap} (B,E)) \tilde{\subseteq} \dot{D}(A,E) \tilde{\cap} \dot{D}(B,E)$. □

Remark(2.4.7) :

For any two soft sets *(A,E)* and *(B,E)* in (X,τ,E,I), then $\dot{D}(A,E)\tilde{\cap}\dot{D}(B,E)\tilde{\not\subset}\dot{D}((A,E)\tilde{\cap}(B,E))$ in general.

Example :

Let X= {a,b} , E = $\{e_1,e_2\}$. Let τ = $\{\phi_E, X_E$, *(C,E)* ,*(D,E)*,(H,E)} where *(C,E)*and *(D,E)* are soft sets such that *(C,E)* = $\{(e_1,\{b\}), (e_2,\phi)\}$, *(H,E)* = $\{(e_1,\{b\}), (e_2,\{b\})\}$ and *(D,E)*= $\{(e_1,\phi),(e_2,\{b\})\}$. let $I=\{\phi_E\}$, *(A,E)*=$\{(e_1,\{a\}), (e_2,\{a,b\})\}$ and *(B,E)*= $\{(e_1,\{a,b\}),(e_2,\{a\})\}$.

Then a$\tilde{\in}$ *(A,E)* and a$\tilde{\in}$ X_E is the only SSIg-open set which containing a and $X_E\tilde{\cap}(A,E)-\{x\}\neq\phi_E$,then a$\tilde{\in}\dot{D}(A,E)$ and a$\tilde{\in}$*(B,E)* and a$\tilde{\in}$ X_E is the only SSIg-open set which containing a and $X_E\tilde{\cap}(B,E)-\{x\}\neq\phi_E$, then a$\tilde{\in}\dot{D}(B,E)$, so a$\tilde{\in}\dot{D}(A,E)\tilde{\cap}\dot{D}(B,E)$. But $(A,E)\tilde{\cap}(B,E)=\{(e_1,\{a\}), (e_2,\{a\})\}$, therefore a$\tilde{\notin}$ $\dot{D}\{(e_1,\{a\}),(e_2,\{a\})\}$, hence a$\tilde{\notin}\dot{D}((A,E)\tilde{\cap}(B,E))$, thus $\dot{D}(A,E)\tilde{\cap}\dot{D}(B,E)\tilde{\not\subset}\dot{D}((A,E)\tilde{\cap}(B,E))$. □

Proposition (2.4.8):

For any two soft sets *(A,E)* and *(B,E)* in (X,τ,E,I), $\dot{D}(\dot{D}(A,E))\tilde{\subseteq}\dot{D}(A,E)$.

Proof:

Let $x\tilde{\notin}\dot{D}(A,E)$, then there exists SSIg-open set *(V,E)* containing x such that $(V,E)\cap(A,E)-\{x\}=\phi_E$, We prove that $x\tilde{\notin}\dot{D}(\dot{D}(A,E))$. Suppose on the contrary that $x\tilde{\in}\dot{D}(\dot{D}(A,E))$. Then for each SSIg-open set *(U,E)* continuing x we have , $(U,E)\tilde{\cap}\dot{D}(A,E)-\{x\}\neq\phi_E$. Therefore there is $y\neq x$ such that $y\tilde{\in}(U,E)\tilde{\cap}\dot{D}(A,E)-\{x\}$. It follows that $y\tilde{\in}\{(U,E)-\{x\}\tilde{\cap}(A,E)-\{y\}\}$. Hence $\{(U,E)-\{x\}\tilde{\cap}(A,E)-\{y\}\}\neq\phi_E$ a contradiction to the fact that $(V,E)\tilde{\cap}(A,E)-\{x\}=\phi_E$, therefore $x\tilde{\notin}\dot{D}(\dot{D}(A,E))$. Thus $\dot{D}(\dot{D}(A,E))\tilde{\subseteq}\dot{D}(A,E)$. □

2.5 SSIg-border set

Definition(2.5.1) :

For any soft subset *(A,E)* in a soft topological space (X,τ,E) with an ideal *I*. An element $x\tilde{\in}(A,E)$ is SSIg-border of *(A,E)*, if every SSIg-open set *(V,E)* continuing *x* intersects with $(A,E)^c$, the set of all SSIg- border elements of *(A,E)*, denoted by $b^*(A,E)$(is an SSIg-border set of *(A,E)*).

Proposition(2.5.2) :

For any soft subset *(A,E)* in a soft topological space (X,τ,E) with an ideal *I*. $b^*(A,E)=(A,E)\tilde{\cap}cl^*(A,E)^c$.

Proof:

We have $x\tilde{\in}b^*(A,E)$ iff $x\tilde{\in}(A,E)$ and for each SSIg-open *(V,E)* continuing *x* such that $(V,E)\tilde{\cap}(A,E)^c\neq\phi_E$, iff $(V,E)\tilde{\not\subset}(A,E)^c$, iff $x\tilde{\notin}int^*(A,E)^c$ by Definition(2.2.1). Iff $x\tilde{\in}X_E\text{-}int^*(A,E)^c$, iff $x\tilde{\in}cl^*(A,E)^c$ by Remark(2.3.19), iff $x\in(A,E)\tilde{\cap}cl^*(A,E)^c$. Thus $b^*(A,E)=(A,E)\tilde{\cap}cl^*(A,E)^c$. □

Proposition(2.5.3) :

For any soft set *(A,E)* in a soft topological space (X,τ,E) with an ideal *I*.

(1) $b^*(\phi_E)=b^*(X_E)=\phi_E$,

(2) $b^*(A,E)=(A,E)-int^*(A,E)$,

(3) $(A,E)-b^*(A,E)=int^*(A,E)$,

Proof:

(1) By Proposition(2.5.2) we get $b^*(\phi_E)=\phi_E\tilde{\cap}cl^*(\phi_E)^c$, $b^*(\phi_E)=\phi_E$. On the other hand $b^*(X_E)=X_E\tilde{\cap}cl^*(X_E)^c=X_E\tilde{\cap}cl^*(\phi_E)=\phi_E$, thus $b^*(\phi_E)=b^*(X_E)=\phi_E$.

(2) By Theorem(2.5.2) and by Remark(2.3.3) then $b^*(A,E)=(A,E)\tilde{\cap}cl^*(A,E)^c=(A,E)-\{cl^*(A,E)^c\}^c=(A,E)-int^*(A,E)$.

(3) By Theorem(2.5.2) (A,E)-b$^*(A,E)=(A,E)$-$\{(A,E)\tilde{\cap} cl^*(A,E)^c\}=(A,E)$ $\tilde{\cap}\{(A,E)\tilde{\cap} cl^*(A,E)^c\}^c=(A,E)\tilde{\cap} int^*(A,E)$ by Remark (2.3.20). Therefore (A,E)-b$^*(A,E)=int^*(A,E)$ by Proposition(2.2.3). □

Corollary (2.5.4):

Let (A,E) be any soft set in a soft topological space (X,τ,E) with an ideal I. Then b$^*(A,E)\tilde{\subseteq}(A,E)$.

Proof:

By Proposition(2.5.2), b$^*(A,E)=(A,E)\tilde{\cap} cl^*(A,E)^c\tilde{\subseteq}(A,E)$, Thus b$^*(A,E)\tilde{\subseteq}(A,E)$. □

Corollary (2.5.5):

Let (M,E) be any soft set in (X,τ,E,I). Then b$^*(M,E)\tilde{\subseteq}$b(M,E).

Proof:

By Propositions(2.5.2),(2.3.15), b$^*(M,E)=(M,E)\tilde{\cap} cl^*(M,E)^c\tilde{\subseteq}(M,E)$ $\tilde{\cap} cl(M,E)^c=$ b(M,E). Thus b$^*(M,E)\tilde{\subseteq}$b(M,E). □

Remark(2.5.6):

The equality in Corollary(2.5.5) need not true in general .

Example :

Let X= {a,b,c} , E = $\{e_1,e_2\}$, I=$\{\phi_E\}$ and τ = $\{\phi_E, X_E$, (A,E) ,(B,E) ,(C,E) ,(D,E),(H,E) }, where (A,E),(B,E),(C,E),(D,E) and (H,E) are soft sets such that (A,E) = $\{(e_1,\{a,b\}),(e_2,\{a\})\}$,$(B,E)$ =$\{(e_1,\{a,b\}),(e_2,\{b\})\}$, (C,E) = $\{(e_1,\{a,b\}),(e_2,X)\}$, (D,E) = $\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$ and (H,E) = $\{(e_1,\{a,b\}),(e_2,\phi)\}$.

Now (D,E)= $\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$, then the SSIg-closed set which containing (D,E) are X_E and $\{(e_1,X),(e_2,\{a,b\})\}$.

Therefore $cl^*(D,E)=X_E\tilde{\cap}\{(e_1,X),(e_2,\{a,b\})\}=\{(e_1,X),(e_2,\{a,b\})\}$.and $(D,E)^c=\{(e_1,\{c\}),(e_2,\{c\})\}$, but $(D,E)^c$ is SSIg-closed set since it is soft closed set then $cl^*(D,E)^c$={(e_1,{c}),(e_2,{c})}, therefore bd$^*(D,E)$= $cl^*(D,E)\tilde{\cap}$ $cl^*(D,E)^c$= {(e_1,{c}) , (e_2,ϕ)}.

On the other hand b$^*(D,E)=(D,E)\tilde{\cap}cl^*(D,E)^c=\phi_E$, Thus bd$^*(D,E)\tilde{\not\subset}$ b$^*(D,E)$. □

Corollary (2.5.7):

Let (A,E) be any soft set in (X,τ,E,I). Then b$^*(A,E)\tilde{\subseteq}$bd(A,E).

Proof:

Follows from Proposition(2.3.15). □

2.6 SSIg-boundary set

Definition(2.6.1) :

For any soft set (A,E) in (X,τ,E,I). An element x∈X is SSIg- boundary of (A,E), if for every SSIg-open set (V,E) continuing x intersects both (A,E) and $(A,E)^c$, the set of all SSIg-boundary elements of (A,E), denoted by bd$^*(A,E)$.

Proposition(2.6.2):

For any soft subset (A,E) in a soft topological space (X,τ,E) with an ideal I. bd$^*(A,E)$= $cl^*(A,E)\tilde{\cap}cl^*(A,E)^c$.

Proof:

We have $x\tilde{\in}$bd$^*(A,E)$ iff for each SSIg-open (V,E) continuing x such that $(V,E)\cap(A,E)\neq\phi_E$ and $(V,E)\tilde{\cap}(A,E)^c\neq\phi_E$, iff $(V,E)\tilde{\not\subset}(A,E)$ and (V,E) $\tilde{\not\subset}(A,E)^c$, iff $x\tilde{\notin}int^*(A,E)$ and $x\tilde{\notin}int^*(A,E)^c$ by Definition(2.2.1). Iff $x\tilde{\in}$ X_E-$int^*(A,E)$ and $x\tilde{\in}$ X_E-$int^*(A,E)^c$, iff $x\tilde{\in}cl^*(A,E)$ by Remark(2.4.24) and $x\tilde{\in}$ $cl^*(A,E)^c$, iff $x\tilde{\in}cl^*(A,E)\tilde{\cap}cl^*(A,E)^c$. Thus bd$^*(A,E)$= $cl^*(A,E)\tilde{\cap}cl^*(A,E)^c$. □

Remark(2.6.3):

bd$^*(A,E)^c$= bd$^*(A,E)$.

Proposition(2.6.4) :

For any soft set *(A,E)* in (X,τ,E,I). Then

(1) bd$^*(\phi_E)$= bd$^*(X_E)=\phi_E$,

(2) bd**(A,E)*= cl^**(A,E)*- int^**(A,E)*,

(3) *(A,E)*-bd**(A,E)*= int^**(A,E)*.

Proof:

(1) By Proposition(2.6.2) we get bd$^*(\phi_E)$= $cl^*(\phi_E)\tilde{\cap}cl^*(\phi_E)^c$ and by Corollary(2.3.8) that bd$^*(\phi_E)=\phi_E$. On the other hand bd$^*(X_E)=cl^*(X_E)\tilde{\cap}$ $cl^*(X_E)^c=cl^*(X_E)\tilde{\cap}cl^*(\phi_E)=\phi_E$. Thus, bd$^*(\phi_E)$= bd$^*(X_E)=\phi_E$.

(2) By Proposition(2.6.2) and by Remark(2.3.20), then bd**(A,E)*= cl^**(A,E)* $\tilde{\cap}cl^*(A,E)^c = cl^*(A,E)\text{-}\{cl^*(A,E)^c\}^c = cl^*(A,E)\text{-}int^*(A,E)$.

(3) By Proposition(2.6.2), *(A,E)*-bd**(A,E)*=*(A,E)*-$\{cl^*(A,E)\tilde{\cap}cl^*(A,E)^c\}$= $(A,E)\tilde{\cap}\{cl^*(A,E)\tilde{\cap}cl^*(A,E)^c\}^c=\{(A,E)\tilde{\cap}int^*(A,E)^c\}\tilde{\cup}\{(A,E)\tilde{\cap}int^*(A,E)\}$ by Remark (2.3.20). Therefore *(A,E)*-bd**(A,E)*= int^**(A,E)* by Theorem (2.2.3). □

Remark (2.6.5) :

Let *(A,E)* and *(B,E)* be any soft sets in (X,τ,E,I). Then bd$^*\{(A,E)\tilde{\cup}(B,E)\}\neq$ bd$^*(A,E)\tilde{\cup}$bd$^*(B,E)$ and bd$^*\{(A,E)\tilde{\cap}(B,E)\}\neq$ bd$^*(A,E)\tilde{\cap}$bd$^*(B,E)$ in general.

Example :

Let X= {a,b,c} , E = $\{e_1,e_2\}$, I=$\{\phi_E\}$ and τ = $\{\phi_E, X_E$, *(A,E)* ,(B,E) ,(C,E) ,(D,E),(H,E) }, where *(A,E)*,*(B,E)*,*(C,E)*,*(D,E)* and *(H,E)* are a soft sets such that *(A,E)*=$\{(e_1,\{a,b\}),(e_2,\{a\})\}$, *(B,E)*=$\{(e_1,\{a,b\}),(e_2,\{b\})\}$, *(C,E)*= $\{(e_1,\{a,b\}),(e_2,X)\}$, *(D,E)* = $\{(e_1,\{a,b\}),(e_2,\{a,b\})\}$ and *(H,E)* = $\{(e_1,\{a,b\}),(e_2,\phi)\}$.

Now for *(A,E)* and *(B,E)*. Then the SSIg-closed sets which contains *(A,E)* are X_E, $\{(e_1,\{a,b\}),(e_2,\{a,c\})\}$, $\{(e_1,X),(e_2,\{a,b\})\}$, $\{(e_1,X),(e_2,\{a,c\})\}$ and

{(e_1,X) , (e_2,{a})}. Therefore, cl^*((A,E)= X_E $\tilde{\cap}$ {(e_1,{a,b}),(e_2,{a,c})} $\tilde{\cap}$ {(e_1,X),(e_2,{a,b})} $\tilde{\cap}$ {(e_1,X),(e_2,{a,c})} $\tilde{\cap}$ {(e_1,X),(e_2,{a})}={(e_1,{a,b}),(e_2,{a})}. And the SSIg-closed set which contains *(B,E)* are X_E,{(e_1,{a,b}), (e_2,{a,b})},{(e_1,X),(e_2,{a,b})},{(e_1,X), (e_2,{b,c})} and {(e_1,X) , (e_2,{b})}. Therefore cl^*((B,E)= X_E $\tilde{\cap}$ {(e_1,{a,b}),(e_2,{a,b})} $\tilde{\cap}$ {(e_1,X),(e_2,{a,b})} $\tilde{\cap}$ {(e_1,X) , (e_2,{b,c})} $\tilde{\cap}$ {(e_1,X) , (e_2,{b})}={(e_1,{a,b}),(e_2,{b})}. And since *(A,E)* $\tilde{\in}$ τ ,hence *(A,E)*^{c} is soft closed set and by Proposition(2.1.3) we get it is SSIg-closed set , therefore by Theorem(2.3.4) , cl^**(A,E)*^{c}=*(A,E)*^{c}= {(e_1,{c}),(e_2,{b,c})}. Hence by Proposition(2.6.2.) bd**(A,E)*= cl^**(A,E)* $\tilde{\cap}$ cl^**(A,E)*^{c}={(e_1,ϕ),(e_2,ϕ)}. Also since *(B,E)* $\tilde{\in}$ τ ,hence *(B,E)*^{c} is soft closed set and by Proposition(2.1.3) we get it is SSIg-closed set , therefore by Proposition(2.4.4) , cl^**(B,E)*^{c}=*(B,E)*^{c}= {(e_1,{c}),(e_2,{a,c})}. Hence by Proposition(2.6.2) bd**(B,E)*= cl^**(B,E)* $\tilde{\cap}$ cl^**(B,E)*^{c}={(e_1,ϕ),(e_2,ϕ)}. Then bd**(A,E)* $\tilde{\cup}$ bd**(B,E)*= {(e_1,ϕ),(e_2,ϕ)} $\tilde{\cup}$ {(e_1,ϕ),(e_2,ϕ)}=ϕ_E .

On the other hand *(A,E)* $\tilde{\cup}$ *(B,E)*= {(e_1,{a,b}),(e_2,{a,b})}. Then the SSIg-closed sets which contains *(A,E)* $\tilde{\cup}$ *(B,E)* are X_E and {(e_1,X) , (e_2,{a,b})}. Therefore cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*}= X_E $\tilde{\cap}$ {(e_1,X) , (e_2,{a,b})}={(e_1,X) , (e_2,{a,b})}. Hence cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*}={(e_1,X) , (e_2,{a,b})}. And since *(A,E)* $\tilde{\cup}$ *(B,E)* $\tilde{\in}$ τ ,hence {*(A,E)* $\tilde{\cup}$ *(B,E)*}^{c} is soft closed set and by Proposition(2.1.3) we get it is SSIg-closed set , therefore by Proposition(2.4.4), cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*}^{c}={*(A,E)* $\tilde{\cup}$ *(B,E)*}^{c}= {(e_1,{c}), (e_2,{c})}. Hence by Proposition(2.6.2) bd*{*(A,E)* $\tilde{\cup}$ *(B,E)*}= cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*} $\tilde{\cap}$ cl^*{*(A,E)* $\tilde{\cup}$ *(B,E)*}^{c}={(e_1,{c}),(e_2,ϕ)}. Thus bd*{*(A,E)* $\tilde{\cup}$ *(B,E)*} $\tilde{\not\subseteq}$ bd**(A,E)* $\tilde{\cup}$ bd**(B,E)*.

Now, to show that, bd*{*(A,E)* $\tilde{\cap}$ *(B,E)*} $\tilde{\not\supseteq}$ bd**(A,E)* $\tilde{\cap}$ bd**(B,E)*.

Then *(A,E)* $\tilde{\cap}$ *(B,E)*= {(e_1,{a,b}),(e_2,ϕ)}, Then the SSIg-closed sets which contains *(A,E)* $\tilde{\cap}$ *(B,E)* are X_E ,{(e_1,X) , (e_2,ϕ)},{(e_1,X) , (e_2,{a})},{(e_1,X) ,

$(e_2,\{b\})\}$. Therefore $cl^*\{(A,E)\tilde{\cap}(B,E)\}= X_E \tilde{\cap}\{(e_1,X), (e_2,\phi)\}\tilde{\cap}\{(e_1,X), (e_2,\{a\})\}\tilde{\cap}\{(e_1,X), (e_2,\{b\})\}=\{(e_1,X), (e_2,\phi)\}$.

Hence $cl^*\{(A,E)\tilde{\cap}(B,E)\}= \{(e_1,X), (e_2,\phi)\}$. And since $(A,E)\tilde{\cap}(B,E)\in\tau$,hence $\{(A,E)\tilde{\cap}(B,E)\}^c$ is soft closed set and by Proposition(2.1.3) we get it is SSIg-closed set , therefore by Proposition(2.3.4), $cl^*\{(A,E)\tilde{\cap}(B,E)\}^c= \{(A,E)\tilde{\cap}(B,E)\}^c= \{(e_1,\{c\}),(e_2,X)\}$. Hence by Proposition(2.6.2) $bd^*\{(A,E)\tilde{\cap}(B,E)\}= cl^*\{(A,E)\tilde{\cap}(B,E)\}\tilde{\cap}cl^*\{(A,E)\tilde{\cap}(B,E)\}^c=\{(e_1,\{c\}),(e_2,\phi)\}$. Thus, $bd^*\{(A,E)\tilde{\cap}(B,E)\}\tilde{\not\subset} bd^*(A,E)\tilde{\cap}bd^*(B,E)$. □

Corollary (2.6.6):

Let (A,E) be any soft set in (X,τ,E,I). Then $bd^*(A,E)\tilde{\subseteq} bd\,(A,E)$.

Proof:

Let $x\tilde{\notin}$ bd (A,E), then $x\tilde{\notin} cl(A,E)$ and $x\tilde{\notin} cl(A,E)^c$ by Proposition(2.3.15), then $x\tilde{\notin} cl^*(A,E)$ and $x\tilde{\notin} cl^*(A,E)^c$, therefore $x\tilde{\notin} bd^*(A,E)$, thus $bd^*(A,E)\tilde{\subseteq} bd\,(A,E)$. □

Remark(2.6.7):

$bd^*(A,E)\tilde{\not\supseteq} bd\,(A,E)$ in general.

Example :

Let X= {a,b,c} , E = $\{e_1,e_2\}$, I=$\{\phi_E\}$ and τ = $\{\phi_E, X_E\}$, let (A,E) be a soft set such that $(A,E) = \{(e_1,\{a,b\}), (e_2,\{a\})\}$. Then the SSIg-closed sets which contains (A,E) are X_E, $\{(e_1,\{a,b\}),(e_2,\{a,c\})\}$, $\{(e_1,X), (e_2,\{a,b\})\}$, $\{(e_1,X), (e_2,\{a,c\})\}$ and $\{(e_1,X), (e_2,\{a\})\}$. Therefore $cl^*((A,E)= X_E \tilde{\cap}\{(e_1,\{a,b\}),(e_2,\{a,c\})\}\tilde{\cap}\{(e_1,X),(e_2,\{a,b\})\}\tilde{\cap}\{(e_1,X),(e_2,\{a,c\})\}\tilde{\cap}\{(e_1,X), (e_2,\{a\})\}=\{(e_1,\{a,b\}), (e_2,\{a\})\}$. And since $(A,E)\in\tau$, hence $(A,E)^c$ is soft closed set and by Proposition(2.1.3) we get it is SSIg-closed set , therefore by Proposition(2.4.4) , $cl^*(A,E)^c=(A,E)^c = \{(e_1,\{c\}),(e_2,\{b,c\})\}$. Therefore, $bd^*(A,E)=cl^*(A,E)\tilde{\cap}cl^*(A,E)^c =\{(e_1,\{a,b\}),(e_2,\{a\})\}\tilde{\cap}\{(e_1,\{c\}),(e_2,\{b,c\})\}= \phi_E$.

On the other hand $cl((A,E)= X_E$ and $cl(A,E)^c = X_E$, therefore bd$(A,E)= cl^*(A,E) \tilde{\cap} cl^*(A,E)^c = X_E$. Thus bd$(A,E) \tilde{\not\subset}$ bd$^*(A,E)$.

Remark (2.6.8):

Let (A,E) be any soft set in (X,τ,E,I). Then b$^*(A,E) \tilde{\subseteq}$ bd$^*(A,E)$.

Proof:

By Proposition(2.6.2),(2.3.2) b$^*(A,E)= (A,E) \tilde{\cap} cl^*(A,E)^c \tilde{\subseteq} cl^*(A,E) \tilde{\cap} cl^*(A,E)^c =$ bd$^*(A,E)$. Thus b$^*(A,E) \tilde{\subseteq}$ bd$^*(A,E)$. □

CHAPTER THREE

SOFT STRONGLY GENERALIZED MAPPING WITH RESPECT TO AN IDEAL IN SOFT TOPOLOGICAL SPACE

In this Chapter we define five different kinds of soft mappings in soft topological spaces with an ideal I, which are SSIg-continuous, contra-SSIg-continuous, SSIg-open, SSIg-closed and SSIg-irresolute mappings, then we shall use them to define the concept of SSIg-homeomorphism .

On the other hand, we studied the composition of any two soft mappings of the same type or of different types, with proofs and examples to disprove.

3.1Basic Properties of mapping in topological space :

"***Definition(3.1.1)***:

A mapping $f : (X,\tau) \to (Y,\sigma)$ is called

(1) *g*-continuous if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V of (Y,σ).

(2) *g*-continuous if $f^{-1}(V)$ is g-open in (X,τ) for every open set V of (Y,σ).

(3) g-open map if $f(U)$ is g-open in (Y,σ) for every open set V of (X,τ).

(4) g-closed map if $f(U)$ is g-closed in (Y,σ) for every closed set V of (X,τ).

(5) contra-g-continuous if $f^{-1}(V)$ is g-closed in (X,τ) for every open set V of (Y,σ)."[8]

"***Theorem(3.1.2):***

Every continuous map is a g-continuous map but not conversely."[8]

"***Lemma(3.1.3):***

Let X, Y and Z be topological spaces, and let $f : X \to Y$ be a g-continuous mapping and $g : Y \to Z$ be a continuous mapping. Then the composition $g_o f: X \to Z$ of the mapping f and g is g-continuous."[8]

"***Definition(3.1.4):***

Let $f : (X,\tau) \to (Y,\sigma)$ be a bijective function. Then the following statement are equivalent

(1) f is g-homeomorphism,

(2) f is g-continuous and f^{-1} is g-continuous,

(3) f is a g-continuous and g-closed mapping,

(4) f is g-continuous and g-open mapping."[8]

3.2 Soft mappings

"***Definition(3.2.1):***

Let SS(X,E) and SS(Y,B) be families of soft sets over X and Y respectively, $u:X\rightarrow Y$ and $p:E\rightarrow B$ be mappings . Then the mapping f_{pu}:SS(X,E) $\rightarrow$SS(Y,B) is defined as :

1- If (F,E)$\in$SS(X,E) , then the image of (F,E) under f_{pu} , written as

f_{pu}(F,E) =(f_{pu}(F),p(E)) is a soft set in SS(Y,B) such that

$$f_{pu}(F)(b)=\begin{cases}\bigcup_{a\in p^{-1}(b)} u(F(a)) \ , & p^{-1}(b)\neq\phi. \\ \phi \ , & p^{-1}(b)=\phi.\end{cases}$$ for all b$\in$B.

2- If (G,B)$\in$SS(Y,B) , then the inverse image of (G,B) under f_{pu} , written as

f_{pu}^{-1}(G,B) =(f_{pu}^{-1}(G),p^{-1}(B)) is a soft set in SS(X,E) , such that

$$f_{pu}^{-1}(G)(a)=\begin{cases}u^{-1}(G(p(a))) \ , & p(a)\in B. \\ \phi \ , & \text{o.w.}\end{cases}$$ for all a$\in$E.

is called a soft mapping , and it is soft bijective if p and u are bijective."[14]

Definition(3.2.2):

Let f_{pu} : SS(X,E)$\rightarrow$SS(Y ,K) and g_{qs} : SS(Y,K)$\rightarrow$SS(Z,H) be a soft mappings . Then $(g\circ f)_{(q\circ p)(s\circ u)}$:SS(X,E)$\rightarrow$SS(Z,H), if (F,E)$\in$SS(X,E), and the image of (F,E) under $(g\circ f)_{(q\circ p)(s\circ u)}$, written as $(g\circ f)_{(q\circ p)(s\circ u)}$ (F,E) =($(g\circ f)_{(q\circ p)(s\circ u)}$ (F),$q\circ p$ (E)) is a soft set.

Remark(3.2.3):

In Definition(3.2.2) $(g \circ f)_{(q\circ p)(s\circ u)} = g_{qs}\ f_{pu}$.

"***Theorem (3.2.4):***

Let f_{pu} : SS(X, E) → SS(Y,K) , $u : X \to Y$, and $p : E \to K$ be mappings. Then for soft sets (F,A) and (F_i, A_i) for $i \in \Lambda$ in SS(X, E) and (G,B) in SS(Y,K) ,we have the following properties :

1- $f_{pu}(\phi_E) = \phi_K$.

2- $f_{pu}(X_E) \tilde{\subseteq} Y_K$.

3- $f_{pu}((F,A)\tilde{\cup}(G,B)) = f_{pu}(F,A) \cup f_{pu}(G,B)$, in general we get

$f_{pu}(\tilde{\cup}_i(F_i,A_i)) = \hat{\cup}_i f_{pu}((F_i,A_i))\ \ \forall i \in \Lambda.$

4- $f_{pu}((F,A)\tilde{\cap}(G,B)) \subseteq f_{pu}(F,A) \cap f_{pu}(G,B)$, in general we get

$f_{pu}(\tilde{\cap}_i(F_i,A_i)) \subseteq \hat{\cap}_i f_{pu}((F_i,A_i))\ \ \forall i \in \Lambda.$

5- If $(F,A) \tilde{\subseteq} (G,B)$, then $f_{pu}(F,A) \tilde{\subseteq} f_{pu}(G,B)$.

6- $f_{pu}^{-1}(\phi_K) = \phi_E$.

7- $f_{pu}^{-1}(Y_K) = X_E$.

8- $f_{pu}^{-1}((F,A)\tilde{\cup}(G,B)) = f_{pu}^{-1}(F,A) \cup f_{pu}^{-1}(G,B)$, in general we get

$f_{pu}(\tilde{\cup}_i(F_i,A_i)) = \hat{\cup}_i f_{pu}((F_i,A_i))\ \ \forall i \in \Lambda..$

9- $f_{pu}^{-1}((F,A)\tilde{\cap}(G,B)) = f_{pu}^{-1}(F,A) \cap f_{pu}^{-1}(G,B)$, in general we get

$f_{pu}^{-1}(\tilde{\cap}_i(F_i,A_i)) = \hat{\cap}_i f_{pu}^{-1}((F_i,A_i))\ \ \forall i \in \Lambda.$"[6]

"***Theorem (3.2.5):***

Let SS(X, E) and SS(Y,K) be two families of soft sets. For a function f_{pu}: SS(X, E) → SS(Y,K) such that u: $X \to Y$, and $p : E \to K$. Then the following statement are true :

$1-$ $f_{pu}^{-1}(G,B)^c = \{f_{pu}^{-1}(G,B)\}^c$ for any soft set (G,B) in SS(Y,K).

$2-f_{pu}(f_{pu}^{-1}(G,B))\tilde{\subseteq}(G,B)$ for any soft set (G,B) in SS(Y,K).

$3-(F,A)\tilde{\subseteq}f_{pu}^{-1}(f_{pu}(F,A))$ for any soft set (F,A) in SS(X,E)."[21]

"***Definition(3.2.6):***

Let (X,τ_X,E) and (Y,τ_Y,B) be two soft topological spaces, $f_{pu}:(X,\tau_X,A)\rightarrow(Y,\tau_Y,B)$ be a mapping. For each soft neighbourhood (H,E) of $(f(x)_e,E)$, if there exists a soft neighbourhood (F,E) of (x_e,E) such that $f_{pu}((F,E))\tilde{\subseteq}(H,E)$, then f_{pu} is said to be soft continuous mapping at (x_e,E). If f_{pu} is soft continuous mapping for all (x_e,E), then f_{pu} is called soft continuous mapping."[14]

"***Theorem (3.2.7):***

(X,τ_X,E) and (Y,τ_Y,B) be two soft topological spaces , $f_{pu}:(X,\tau_X,E)\rightarrow(Y,\tau_Y,B)$ be a mapping. Then the following conditions are equivalent:

(1) $f_{pu}:(X,\tau_X,E)\rightarrow(Y,\tau_Y,B)$ is a soft continuous mapping,

(2) For each soft open set (G,E) over Y, $f_{pu}^{-1}((G,E))$ is a soft open set over X,

(3) For each soft closed set (H,E) over Y, $f_{pu}^{-1}((H,E))$ is a soft closed set over X,

(4) For each soft set (F,E) over X, $f_{pu}(cl(F,E))\tilde{\subseteq}cl(f_{pu}(F,E))$,

(5) For each soft set (F,E) over X, $int(f_{pu}(F,E))\tilde{\subseteq}f_{pu}(int(F,E))$,

(6) For each soft set (G,E) over Y , $cl(f_{pu}^{-1}(G,E))\tilde{\subseteq}f_{pu}^{-1}(cl(G,E))$,

(7) For each soft set (G,E) over Y , $f_{pu}^{-1}(int(G,E))\tilde{\subseteq}int(f_{pu}^{-1}(G,E))$."[14]

"***Definition (3.2.8):***

Let (X,τ_X,E) and (Y,τ_Y,B) be two soft topological spaces, $f_{pu}:(X,\tau_X,E)\rightarrow(Y,\tau_Y,B)$ be a mapping.

(1) If the image f_{pu}((F,E)) of each soft open set (F,E) over X is a soft open set in Y , then f_{pu} is said to be a soft open mapping.

(2) If the image f_{pu}((H,E)) of each soft closed set (H,E) over X is a soft closed set over Y , then f_{pu} is said to be a soft closed mapping."[6],[14]

"***Theorem (3.2.9):***

Let (X,τ_X,E) and (Y,τ_Y,E) be two soft topological spaces, f_{pu} : (X,τ_X,E) $\to$ (Y, τ_Y ,E) be a mapping. Then f_{pu} is a soft closed mapping if and only if for each soft set (F,E) over X, $cl(f_{pu}(F,E)) \tilde{\subseteq} f_{pu}(cl(F,E))$ is satisfied."[14]

"***Theorem (3.2.10):***

Let (X,τ_X,E) and (Y,τ_Y,E) be two soft topological spaces, f_{pu} : (X,τ_X,E) $\to$ (Y,τ_Y,E) be a mapping. Then f_{pu} is a soft open mapping if and only if for each soft set (F,E) over X, $f_{pu}(int(F,E)) \tilde{\subseteq} int(f_{pu}(F,E))$ is satisfied."[14]

"***Definition(3.2.11):***

Let (X,τ_X,E) and (Y,τ_Y,E) be two soft topological spaces, f_{pu} : (X,τ_X,E) $\to$ (Y, τ_Y ,E) be a soft mapping. If f_{pu} is a bijection, soft continuous and f_{pu}^{-1} is a soft continuous mapping, then f_{pu} is said to be soft homeomorphism from X to Y . When a soft homeomorphism f_{pu} exists between X and Y , we say that X is soft homeomorphic to Y."[14]

"***Theorem (3.2.12):***

Let (X,τ_X,E) and (Y,τ_Y,E) be two soft topological spaces, f_{pu}: $(X,\tau_X,E) \rightarrow (Y,\tau_Y,E)$ be a soft bijective mapping. Then the following conditions are equivalent:

(1) f_{pu} is a soft homeomorphism,

(2) f_{pu} is a soft continuous and soft closed mapping,

(3) f_{pu} is a soft continuous and soft open mapping."[14]

"***Note(3.2.13):***

If (X,τ,I) is a topological space with an ideal I , (Y,σ) is a topological space and $f : (X,\tau,I) \rightarrow (Y,\sigma)$ is a function, then $f(I) = \{f(I_i) : I_i \in I , \forall i \in \Lambda \}$ is an ideal of Y. So in this chapter we will use I as an ideal over X and $f(I)$ is an ideal over Y."[6]

"***Definition (3.2.14):***

Let (X,τ_X,E,I) and (Y,τ_Y,E) be two soft topological spaces, f_{pu}: $SS(X,E) \rightarrow SS(Y,E)$ be a soft mapping. then f_{pu} is said to be soft Ig-continuous if the inverse image under f_{pu} of every soft open set in SS(Y,E) is soft Ig-open set in SS(X,E)."[14]

"***Definition(3.2.15):***

Let (X,τ_X,E,I) and (Y,τ_Y,E) be two soft topological spaces, f_{pu}: $(X,\tau_X,E,I) \rightarrow (Y, \tau_Y ,E)$ be a mapping.

(1) If the image $f_{pu}((F,E))$ of each soft Ig-open set (F,E) over X is a soft $f(I)$g-open set in SS(Y,E) , then f_{pu} is said to be a soft Ig-open mapping.

(2) If the image f_{pu}((H,E)) of each soft Ig-closed set (H,E) over X is a soft *f*(I)g-closed set in SS(Y,E) , then f_{pu} is said to be a soft Ig-closed mapping."[14]

3.3 SSIg- Continuity

Definition(3.3.1) :

Let (X,τ_X,*A*,*I*) and (Y,τ_Y,B) be two soft topological spaces with an ideal *I*, f_{pu} : (X, τ_X,A) $\rightarrow$ (Y, τ_Y ,B) be a mapping. If for each soft open set (G,B) over Y , f_{pu}^{-1}((G,B)) is a SSIg-open set over X, then f_{pu} is said to be SSIg-continuous mapping.

Example(3.3.2) :

Let X={a,b,c} ,E = {e_1, e_2} , Y={h_1, h_2, h_3}, K = {k_1, k_2}, $I=\{\phi\}$, and τ = { ϕ_E , X_E , (F,E)}, ϑ = { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{ b,c }), (e_2,{ a })} , (G,K)= {(k_1,{ h_3,h_2 }), (k_2,{ h_1 })}. Define $p:E\rightarrow K$ *such that* $p(e_2)=k_2$, $p(e_1)=k_1$ and $u:X\rightarrow Y$ *such that* $u(a)=h_1, u(b)=h_3, u(c)=h_2$.

Then , f_{pu} : (X,τ,E,I) $\rightarrow$(Y,ϑ,K) is a soft mapping and it is an SSIg-continuous . Since , the soft open sets in (Y,ϑ,K) are ϕ_K , Y_K and (G,K) ,then $f_{pu}^{-1}(\phi_K)=\phi_E$ is a SSIg-open , $f_{pu}^{-1}(Y_K)=X_E$ is a SSIg-open.

$f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_3,h_2\}),(k_2,\{h_1\})\})$ ={(e_1,{ b,c }), (e_2,{ a })}= (F,E).

(F,E)^c={(e_1,{ a }), (e_2,{ b,c })}, *int(F,E)*^{c} $=\phi_E$, then *cl(int(F,E)*^{c}*)* $=\phi_E$, since X_E is soft open set in (X,τ,E) which contains (F,E)^c and *cl(int(F,E)*^{c}*)* $=\phi_E$. So *cl(int(F,E)*^{c}*)*- $X_E \in I$. Hence , (F,E)^c is a SSIg-closed. Therefore, (F,E) is an SSIg-open . □

Proposition (3.3.3):

Every soft continuous mapping is SSIg-continuous mapping .

Proof:

Let $f_{pu} : (X,\tau,E,I) \to (Y, \vartheta, K)$ be a soft continuous mapping.

Let (H,K) be a soft open set in (Y, ϑ, K), since f_{pu} is a soft continuous mapping, then $f_{pu}^{-1}(H,K)$ is soft open set, so $(f_{pu}^{-1}(H,K))^c$ is soft closed set. But we have every soft closed set is SSIg-closed from Proposition(2.1.3), then $(f_{pu}^{-1}(H,K))^c$ is SSIg-closed, hence $f_{pu}^{-1}(H,K)$ is SSIg-open set, thus f_{pu} is a SSIg-continuous mapping. □

Remark (3.3.4):

SSIg-continuous mapping need not to be a soft continuous mapping in general.

Example:

Let X={a,b,c} ,E = {e_1, e_2} , Y={h_1, h_2, h_3}, K = {k_1, k_2}, $I = \{\phi_E\}$ and $\tau = \{\phi_E, X_E\}$, $\vartheta = \{\phi_K, Y_K, (G,K)\}$ be two soft topologies defined on X and Y respectively , where (G,E)={($k_1, \{h_1\}$), ($k_2, \{h_3, h_2\}$)}. Define $p: E \to K$ *such that* $p(e_1) = k_2$, $p(e_2) = k_1$ and $u: X \to Y$ *such that* $u(a) = h_3, u(b) = h_2, u(c) = h_1$.

Then , f_{pu} : (X,τ,E,I) →(Y, ϑ ,K) is a soft mapping and it is a SSIg-continuous .

Since , the soft open sets in (Y, ϑ ,K) are ϕ_k , Y_K and (G,K) ,then $f_{pu}^{-1}(\phi_K) = \phi_E$ is a SSIg-open, $f_{pu}^{-1}(Y_K) = X_E$ is a SSIg-open. $f_{pu}^{-1}((G,K)) = f_{pu}^{-1}(\{(k_1, \{h_3, h_2\}), (k_2, \{h_1\})\})$ ={($e_1, \{c\}$), ($e_2, \{a,b\}$)}= (H,E), *int*(H,E)^c $= \phi_E$, then *cl(int*(H,E)^c) $= \phi_E$, since X_E is soft open set in (X,τ,E) which contains (F,E)^c and *cl(int(F,E)*^{c}*)* $= \phi_E$. So *cl(int(F,E)*^{c}*)*- $X_E \in I$. Hence , (H,E)^c is a SSIg-closed. Therefore, (H,E) is a SSIg-open so, f_{pu} is SSIg-continuous but it is not soft continuous since (G,K) is soft open set in (Y, ϑ ,K) but $f_{pu}^{-1}((G,K))=$ (H,E) which is not soft open set in (X,τ,E,I) . Therefore f_{pu} is not soft continuous . □

Definition(3.3.5) :

Let (X,τ_X,A,I) and (Y,τ_Y,B) be two soft topological spaces , f_{pu} : (X,τ_X,A,I) → (Y,τ_Y,B) be a mapping. If for each soft open set (G,B) over Y , f_{pu}^{-1}((G,B)) is a SSIg-closed set over X, then f_{pu} is said to be contra-SSIg-continuous mapping. If f_{pu}^{-1}((G,B)) is a soft closed set over X, then f_{pu} is said to be soft contra-continuous mapping.

Proposition(3.3.6):

Every soft contra-continuous mapping is contra-SSIg-continuous .

Proof:

Let $f_{pu}:(X,\tau,E,I)\to$(Y, ϑ ,K) be a soft contra-continuous mapping. Let (H,K) be a soft open set in (Y, ϑ ,K), since f_{pu} is a soft contra-continuous mapping. then $f_{pu}^{-1}(H,K)$ is soft closed set. But we have every soft closed set is SSIg-closed from Proposition(2.1.3) , then $f_{pu}^{-1}(H,K)$ is SSIg-closed, thus f_{pu} is a contra-SSIg-continuous mapping. □

Remark(3.3.7):

contra-SSIg-continuous mapping $\not\Rightarrow$ soft contra-continuous mapping in general.

Example :

Let X={a,b,c} ,E = {e_1, e_2}, Y={h_1, h_2, h_3}, K = {k_1, k_2},I ={ϕ_E} and τ = { ϕ_E , X_E }, ϑ = { ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (G,K) ={(k_1,{ h_1 }), (k_2,{ h_3,h_2 })}. define

$p:E \to K$ *such that* $p(e_1)=k_2$, $p(e_2)=k_1$ and $u:X \to Y$ *such that* $u(a)=h_3, u(b)=h_2, u(c)=h_1$.

Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping and it is a SSIg-continuous . Since , the soft open sets in (Y,ϑ,K) are ϕ, Y_K and (G,K) ,then $f_{pu}^{-1}(\phi_K)=\phi_E$ is a SSIg-closed, $f_{pu}^{-1}(Y_K)=X_E$ is a SSIg- closed .

$f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_3,h_2\}),(k_2,\{h_1\})\})$ $=\{(e_1,\{c\}),(e_2,\{a,b\})\}=$ (H,E), *int*(H,E) $=\phi_E$, then *cl*(*int*(H,E)) $=\phi_E$, since X_E is soft open set in (X,τ,E,I) which contains (H,E) and *cl(int(H,E))* $=\phi_E$. So *cl(int(H,E))*-$X_E \in I$. Hence , (H,E) is a SSIg-closed . Therefore, $f_{pu}^{-1}((G,K))$ is a SSIg-closed set, thus f_{pu} is contra-SSIg-continuous.

But it is not soft contra-continuous since (G,K) is soft open set in (Y,ϑ ,K) but $f_{pu}^{-1}((G,K))=\{(e_1,\{c\}),(e_2,\{a,b\})\}$ is not soft closed set in (X,τ,E,I) . Therefore f_{pu} is not soft contra-continuous . □

Remark(3.3.8):

The concepts of contra-SSIg-continuous and SSIg-continuous are independent by the following examples.

Example :

Let X={a,b,c} ,E = {e_1, e_2} , Y={h_1, h_2, h_3}, K = {k_1, k_2},$I=\{\phi_E\}$ and τ = {ϕ_E , X_E ,(F,E) }, ϑ = {ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)= $\{(e_1,\{a\}),(e_2,\{b,c\})\}$,(G,K)=$\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\}$. Define $p:E \to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X \to Y$ *such that* $u(a)=h_1, u(b)=h_3, u(c)=h_2$.

Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping and the soft open sets in (Y,ϑ,B) are ϕ_K , Y_K and (G,K) ,then $f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\})$ $=\{(e_1,\{a\}),(e_2,\{b,c\})\}=$ (F,E), then *int*(F,E)= (F,E) , then *cl*(*int*(F,E))

$=X_E$, therefore $cl(int$(F,E))- (F,E) $\notin I$. Hence , (F,E) is not SSIg-closed . Thus f_{pu} is not contra-SSIg-continuous.

On the other hand, since $f_{pu}^{-1}((G,K))$ = (F,E) which is soft open set and so it is SSIg-open set by Corollary(2.2.2). Therefore f_{pu} is SSIg-continuous.□

Example:

Let X={a,b,c} ,E = {e_1, e_2}, Y={h_1, h_2, h_3} , K = {k_1, k_2} , $I=\{\phi_E\}$ and τ = {ϕ_E , X_E ,(F,E) }, ϑ = {ϕ_K , Y_K , (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)= $\{(e_1,\{b,c\}),(e_2,\{a\})\}$,(G,K)= $\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\}$. Define $p:E\to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\to Y$ *such that* $u(a)=h_1, u(b)=h_3, u(c)=h_2$.

Then , f_{pu} : (X,τ,E,I) →(Y, ϑ ,K) is a soft mapping and the soft open sets in (Y, ϑ ,K) are ϕ_K , Y_K and (G,K) ,then $f_{pu}^{-1}(\phi_K)=\phi_E$ is a SSIg-closed , $f_{pu}^{-1}(Y_K)=X_E$ is a SSIg- closed and $f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\}=$ $\{(e_1,\{a\}),(e_2,\{b,c\})\}=$ (F,E) which is soft closed set in (X,τ,E,I), therefore by Proposition(2.1.3) we get that $f_{pu}^{-1}((G,K))$ is SSIg-closed set, thus f_{pu} is contra-SSIg-continuous.

But it is not SSIg-continuous since (G,K) is soft open set in (Y, ϑ ,K) but $f_{pu}^{-1}((G,K))=f_{pu}^{-1}(\{(k_1,\{h_1\}),(k_2,\{h_3,h_2\})\})$ $=\{(e_1,\{a\}),(e_2,\{b,c\})\}=$ (F,E), then $cl(int$(F,E)^c) $=X_E$, $cl(int$(F,E)^c)- (F,E)^c $\notin I$. Hence , (F,E)^c is not SSIg-closed , therefore $f_{pu}^{-1}((G,K))$ is not SSIg-open set, hence f_{pu} is not SSIg-continuous . Thus f_{pu} is contra-SSIg-continuous but it is not SSIg-continuous. □

Proposition(3.3.9) :

Let $f_{pu}:(X,\tau,E,I)\to(Y,\vartheta,K)$ is a soft closed mapping. If (G,E) is a soft closed set in (X,τ,E,I) , then $f_{pu}(G,E)$ is SS f_{pu}(I)g-closed in (Y, ϑ ,K).

Proof :

Suppose that (G,E) is a closed SSIg-closed in (X,τ,E,I) .

Let (H,K) be a soft open set in (Y, ϑ ,K) such that $f_{pu}(G,E)\tilde{\subseteq}$(H,K) , then $f_{pu}(G,E)$ is soft closed set in (Y, ϑ ,K) and by Corollary(2.1.10) we get that $f_{pu}(G,E)$ is SS $f_{pu}(I)$ g-closed. □

Corollary(3.3.10):

Let f_{pu} : (X,τ,E,I) →(Y, ϑ ,K) is a soft open mapping . If (G,E) is a soft open set in (X,τ,E,I) , then $f_{pu}(G,E)$ is SS f_{pu}(I)g-open in (Y, ϑ ,K) .

Proof :

It is clear . □

Proposition(3.3.11) :

Let $f_{pu}:(X,\tau,E,I)\to(Y,\vartheta,K)$ is a soft continuous closed mapping. If (G,E) is a soft closed set in (X,τ,E,I) and $I=\{\emptyset_E\}$, then $f_{pu}(G,E)$ is SS f_{pu}(I)g-closed in (Y, ϑ ,K).

Proof:

Let (H,K) be a soft open set in (Y, ϑ ,K) such that $f_{pu}(G,E)\tilde{\subseteq}$(H,K) , then $(G,E)\tilde{\subseteq} f_{pu}^{-1}$(H,K). Therefore $cl(int(G,E)) - f_{pu}^{-1}$(H,K)∈I since f_{pu} is soft continuous and (G,E) is SSIg-closed set over X, then $cl(int(G,E))\tilde{\subseteq}$ f_{pu}^{-1}(H,K) since $I=\{\emptyset_E\}$. Therefore $cl(intf_{pu}(G,E)) -$ (H,K)∈ $f_{pu}(I)$ since f_{pu} is soft closed mapping. Thus $f_{pu}(G,E)$ is SS $f_{pu}(I)$ g-closed set. □

Proposition(3.3.12) :

Let $f_{pu}:(X,\tau,E,I)\to(Y,\vartheta,K)$ is an SSIg-continuous mapping. If Y⊆X, then $f_{pu}|_Y$ is an SSI$_Y$g-continuous.

Proof:

Let (H,K) be a soft open set in (Y, ϑ,K) , then f_{pu}^{-1}(H,K) is SSIg − open set over X . Then $(f_{pu}|_Y)^{-1}(H,K)$is an SSI$_Y$g-open by Theorem(2.1.20). Thus $f_{pu}|_Y$ is an SSI$_Y$g-continuous. □

Proposition(3.3.13) :

Let $f_{pu}:(X,\tau,E,I)\rightarrow$(Y, ϑ ,K) is a soft mapping. If Y⊆X and $f_{pu}|_{Y_E}$ is an SSI$_Y$g-continuous and Y_E is an SSIg-closed, then f_{pu} is an SSIg-continuous mapping.

Proof:

Let (H,K) be a soft open set in (Y, ϑ ,K) , then $(f_{pu}|_Y)^{-1}(H,K)$is an SSI$_Y$g-open, then{ $(f_{pu}|_Y)^{-1}(H,K)\}^c$ SSI$_Y$g-closed set. Therefore f_{pu}^{-1}(H,K) is SSIg − open set over X by Theorem(2.1.21). Thus f_{pu} is an SSIg-continuous mapping. □

Proposition (3.3.14):

Every soft g-continuous mapping is SSIg-continuous.

Proof:

Let f_{pu}: (X,τ,E,I) →(Y, ϑ ,K) be a soft g-continuous mapping. Let (H,K) be a soft open set in (Y, ϑ ,K), since f_{pu} is a soft g-continuous mapping. then $f_{pu}^{-1}(H,K)$ is soft g-open set. But we have every soft g-open set is SSIg-open from Corollary(2.1.10) , then $f_{pu}^{-1}(H,K)$ is SSIg-open, thus f_{pu} is a SSIg-continuous mapping. □

Remark(3.3.15):

SSIg-continuous $\not\Rightarrow$ soft g-continuous in general.

Example:

Let X={a,b,c} ,E = {e_1, e_2} , Y={h_1, h_2, h_3}, K = {k_1, k_2}, $I=\{\phi_E\}$, and τ = {ϕ_E, X_E ,(F,E) }, ϑ = {ϕ_K, Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{a,c}), (e_2,{a,b})} and (G,K)={(k_1,{h_1,h_2}), (k_2,{h_3})}. Define $p:E\to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\to Y$ *such that* $u(a)=h_1, u(b)=h_3, u(c)=h_2$.

Then , f_{pu} : (X,τ,E,I) →(Y, ϑ ,K) is a soft mapping and it is a SSIg-continuous .

But it is not soft g-continuous since (G,K) is soft open set in (Y, ϑ ,K) since (V,E)^c $\tilde{\subseteq}$ (F,E) but cl(V,E)^c= X_E $\tilde{\not\subset}$ (F,E). Hence $f_{pu}^{-1}((G,K))$ is not soft g-open set. □

Proposition(3.3.16) :

Let f_{pu} : (X,τ,E,I) →(Y, ϑ ,K) be an SSIg-continuous mapping and g_{qs} : (Y, ϑ ,K) →(Z, η ,H) is a soft continuous mapping . Then $g_{qs}\circ f_{pu}$: (X,τ,E,I)→(Z, η ,H) is SSIg-continuous mapping .

Proof :

Let f_{pu} : (X,τ,E,I) →(Y, ϑ ,K) is a SSIg-continuous mapping and g_{pu} : (Y, ϑ ,K) →(Z, η ,H) is a soft continuous mapping. To prove that $(g\circ f)_{pu}$: (X,τ,E,I)→(Z, η ,H) is SSIg-continuous mapping .

Let (M,H) be a soft open set in (Z, η ,H) . Since g_{qs} is a soft continuous mapping . Then $g_{qs}^{-1}(M,H)$ is soft open set in (Y, ϑ ,K) and since f_{pu} is SSIg-continuous mapping and $g_{qs}^{-1}(M,H)$ is soft open set in (Y, ϑ ,K), So $f_{pu}^{-1}(g_{qs}^{-1}(M,H))$ is SSIg-open set in (X,τ,E,I) . Then $(g_{qs}\circ f_{pu})^{-1}(M,H)$ = $f_{pu}^{-1}(g_{qs}^{-1}(M,H))$. Hence, $g_{qs}\circ f_{pu}$ is SSIg-continuous mapping . □

Remark(3.3.17) :

If $f_{pu}:(X,\tau,E,I)\to(Y,\vartheta,K)$ is a SSIg-continuous mapping and $g_{qs}:(Y,\vartheta,K)\to(Z,\eta,H)$ is a SSIg-continuous mapping . Then $g_{qs}\circ f_{pu}$: $(X,\tau,E)\to(Z,\eta,H)$ need not to be SSIg-continuous.

Example:

Let X={a,b,c} , Y={d,e,s}, Z={r,m,n}, E={e_1,e_2}, K={k_1,k_2}, H={h_1,h_2}, I={ϕ_E} and let τ_X={ϕ_E, X_E,(F,E)}, τ_Y={ϕ_K, Y_K}, τ_Z={ϕ_H, Z_H,(G,H)} be a soft topologies defined on X , Y and Z respectively , where (F,E), (G,H) are:

(F,E)={(e_1,{a,b}),(e_2,{c})}, (G,H)={(h_1,{n}), (h_2,{r,m})}.

Define $f_{pu}:(X,\tau_X,E,I)\to(Y,\tau_Y,K)$ and $g_{q\omega}:(Y,\tau_Y,K)\to(Z,\tau_Z,H)$ such that $p:E\to K$ *such that* $p(e_1)=k_2$, $p(e_2)=k_1$, $q:K\to H$ *such that* $q(k_1)=h_1$, $q(k_2)=h_2$, $u:X\to Y$ *such that* $u(a)=d, u(b)=s, u(c)=e$ and $\omega:Y\to Z$ *such that* $\omega(d)=r$, $\omega(e)=n$, $\omega(s)=m$.

Now, $g^{-1}_{q\omega}(G,H)=$ {(k_1,{e}), (k_2,{d,s})}, put (V,K)= {(k_1,{e}), (k_2,{d,s})} ,then $(V,K)^c$= {(k_1,{d,s}), (k_2,{e})} we need to show that $(V,K)^c$ is SSIg-closed set in (Y,τ_Y,K) , since the only soft open set in (Y,τ_Y,K) is Y_K which contains $(V,K)^c$, so $int(V,K)^c=\phi_K$ and $cl(int(V,K)^c))=\phi_K$, therefore $cl(int(V,K)^c))$-$Y_K\in$ I, hence $(V,K)^c$ is SSIg-closed set in (Y,τ_Y,K). Thus $g_{q\omega}$ is SSIg-continuous.

Now, since the only soft open sets in (Y,τ_Y,K) are ϕ and Y_K, then $f^{-1}_{pu}(Y_K)=X_E$ and $f^{-1}_{pu}(\phi_K)=\phi_E$ which are SSIg-open sets in (X,τ_X,E,I), hence f_{pu} is SSIg-continuous.

Now $g_{q\omega}\circ f_{pu}:(X,\tau_X,E,I)\to(Z,\tau_Z,H)$, $(g_{q\omega}\circ f_{pu})^{-1}$(G,H)= $f^{-1}_{pu}(g^{-1}_{q\omega}(G,H))$ = f^{-1}_{pu}({(k_1,{e}), (k_2,{d,s})})={(e_1,{c}), (e_2,{a,b})}, put (L,E)={(e_1,{c}), (e_2

,{a,b})}, then (L,E)^c={(e_1,{a,b}), (e_2,{c})} and *int*(L,E)^c=(L,E)^c since (L,E)^c is a soft open set, then *cl*(*int*(L,E)^c)= X_E, therefore *cl*(*int*(L,E)^c)- (L,E)^c$\notin I$. Hence $\left(g_{q\omega}\circ f_{pu}\right)^{-1}$(G,H) is not SSIg-open set , thus $g_{q\omega}\circ f_{pu}$ is not SSIg-continuous mapping. □

Remark(3.3.18) :

If f_{pu} : (X,τ,E,I)→(Y,ϑ,K) is a continuous mapping and g_{qs} : (Y,ϑ,K) →(Z,η,H) is a SSIg-continuous mapping. Then $g_{qs}\circ f_{pu}$:(X,τ,E,I)→(Z,η ,H) need not to be SSIg -continuous mapping we can see this from Example of Remark(3.3.14).

Proposition(3.3.19) :

Let f_{pu} : (X,τ,E,I)→(Y,ϑ,K) be a SSIg-continuous mapping and g_{qs} : (Y,ϑ,K)→(Z,η,H) be a soft contra-continuous mapping . Then $g_{qs}\circ f_{pu}$:(X,τ,E,I)→(Z,η,H) is contra-SSIg-continuous mapping .

Proof :

Let f_{pu} : (X,τ,E,I)→(Y,ϑ,K) be a SSIg-continuous mapping and g_{pu} : (Y,ϑ,K)→(Z,η,H) is a soft contra-continuous mapping . To prove that $g_{qs}\circ f_{pu}$: (X,τ,E,I)→ (Z,η,H) is contra-SSIg-continuous mapping .

Let (M,H) be a soft open set in (Z,η,H) . Since g_{qs} is a soft contra-continuous mapping . Then $g_{qs}^{-1}(M,H)$ is soft closed set in (Y,ϑ,K) and since f_{pu} is SSIg-continuous mapping and $g_{qs}^{-1}(M,H)$ is soft closed set in (Y,ϑ,K), therefore $f_{pu}^{-1}(g_{qs}^{-1}(M,H))$ is an SSIg-closed set in (X,τ,E,I). Hence $(g_{qs}\circ f_{pu})^{-1}(M,H)=f_{pu}^{-1}(g_{qs}^{-1}(M,H))$. Thus, $g_{qs}\circ f_{pu}$ is contra -SSIg-continuous mapping . □

Theorem(3.3.20) .

Let f_{pu} : (X,τ,E,I) $\rightarrow$(Y, ϑ,K) be a mapping from a soft space (X,τ,E,I) to a soft space (Y, ϑ,K). If f_{pu} is SSIg-continuous mapping then for each soft singleton (P,E) in X and each soft open set (O,K) in Y and $f_{pu}(P,E)\tilde{\subseteq}(O,K)$, there exists a SSIg-open set (U,E) in X such that $(P,E)\tilde{\subseteq}(U,E)$ and $f_{pu}(U,E)\tilde{\subseteq}(O,K)$.

Proof :

Suppose that f_{pu} is SSIg-continuous mapping .

Let (P,E) be a soft singleton in X and (O,K) be a soft open set in Y such that $f_{pu}(P,E)\tilde{\subseteq}(O,K)$. Then $(P,E)\tilde{\subseteq}f_{pu}^{-1}(O,K)$,but f_{pu} is SSIg-continuous mapping and (O,K) be a soft open set in Y . By definition of SSIg-continuous mapping we get that $f_{pu}^{-1}(O,K)$ is SSIg-open set in X . Put (U,E)= $f_{pu}^{-1}(O,K)$. Therefore , $(P,E)\tilde{\subseteq}(U,E)$ and $(O,K)\tilde{\subseteq}f_{pu}(U,E)$.□

Remark(3.3.21):

The converse of Theorem (3.3.20) is not true in general.

Example :

Let X={a,b} ,E = {e$_1$, e$_2$} , , Y={d, c}, K = {k$_1$, k$_2$}, $I=\{\phi_E\}$, and τ = { ϕ_E, X_E ,(F,E)}, ϑ = { ϕ_K , Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{b}), (e_2,ϕ)} , (G,K)={(k_1,{d}), (k_2,Y)} define $p{:}E\rightarrow K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u{:}X\rightarrow Y$ *such that* $u(a)=d$,$u(b)=c$. Then , f_{pu} : (X,τ,E,I) $\rightarrow$(Y, ϑ,K) is a soft mapping .

Since (G,K) is SS$f_{pu}(I)$g-open and $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$({(k_1,{d}), (k_2,Y)}) = {(e_1,{a}), (e_2,X)} which is not SSIg-open set. So it is not SSIg-continuous mapping.

On the other hand a$\in$X and $f_{pu}(a,E)\tilde{\subseteq}(G,K)\in\tau_Y$ and a$\tilde{\in}$ {(e_1,{a}), (e_2,{a})} where {(e_1,{a}), (e_2,{a})} is SSIg-open set over X and $(a,E)\tilde{\subseteq}\{(e_1,\{a\}),(e_2,\{a\})\}$, $f_{pu}\{(e_1,\{a\}),(e_2,\{a\})\}\tilde{\subseteq} f_{pu}(G,K)$. Also $b\in X$ and $f_{pu}(b,E)\tilde{\subseteq}Y_K$, while X_E is SSIg-open set and $(b,E)\tilde{\subseteq}X_E$, $f_{pu}(X_E)\tilde{\subseteq}Y_K$.□

Proposition(3.3.22) :

Let f_{pu} : (X,τ,E,I) →(Y,ϑ,K) be a mapping from a soft space (X,τ,E) with an ideal I to soft space (Y,ϑ,K).Then the following statements are equivalent :

1- f_{pu} is SSIg-continuous mapping .

2- the inverse image under f_{pu} for any soft closed set over Y is SSIg-closed set over X .

proof :

$(1)\Rightarrow(2)$

suppose that f_{pu} is SSIg-continuous mapping .

To prove that the inverse image under f_{pu} for any SSIg-closed set over Y is SSIg-closed set over X . Let (F,K) be a soft closed set over Y .We have to show that f_{pu}^{-1}(F,K) is SSIg-closed over X . Since (F,K)$\in\vartheta^c$, then (F,K)^c)$\in\vartheta$. Because f_{pu} is SSIg-continuous mapping .

Then f_{pu}^{-1}(F,E)^c is SSIg-open over X and by Theorem(3.2.6) we have f_{pu}^{-1}(F,K)^c = (f_{pu}^{-1}(F,K))^c . Hence, f_{pu}^{-1}(F,K) is SSIg-closed over X .

$(2)\Rightarrow(1)$

Suppose that the inverse image under f_{pu} of any soft closed set over Y is SSIg-closed set over X and to prove that f_{pu} is SSIg-continuous mapping . Let (F,K) be a soft open set over Y .We have to show that f_{pu}^{-1}(F,K) is SSIg-open set in X . Since (F,K) is a soft open set over Y , then (F,K)^c is a SSIg-closed set over Y . Then f_{pu}^{-1}(F,K)^c is SSIg-closed set over X and f_{pu}^{-1}(F,K)^c

= (f_{pu}^{-1}(F,K))^c. Hence, f_{pu}^{-1}(F,K) is SSIg-open set over X . Therefore, f_{pu} is SSIg-continuous mapping . □

In general topology, it is Known that (f is continuous if and only if $cl(f^{-1}(A,E)) \tilde{\subseteq} f^{-1}(cl(A,E))$.

Proposition(3.3.23) :

Let f_{pu} : (X,τ,E,I) →(Y, ϑ,K) be an SSIg-continuous mapping. If (A,E) is any soft set over X , then $f_{pu}(cl^*(A,E)) \tilde{\subseteq} cl(f_{pu}(A,E))$.

Proof :

Let (A,E) be any soft set over X . Then $f_{pu}(A,E)$ is a soft set over Y and $cl(f_{pu}(A,E))$ is soft closed set over Y. But f_{pu} is a SSIg-continuous mapping . Then $f_{pu}^{-1}(cl^*(f_{pu}(A,E)))$ is a SSIg-closed set over X . Then $cl^*(f_{pu}^{-1}(cl(f_{pu}(A,E))) = f_{pu}^{-1}(cl(f_{pu}(A,E)))$ by Theorem(2.3.4).

Then $(A,E) \tilde{\subseteq} f_{pu}^{-1}((f_{pu}(A,E)) \tilde{\subseteq} f_{pu}^{-1}(cl(f_{pu}(A,E)))$ *and* $(A,E) \subseteq f_{pu}^{-1}(cl(f_{pu}(A,E)))$

Therefore $cl^*(A,E) \tilde{\subseteq} cl^*(f_{pu}^{-1}(cl(f_{pu}(A,E))) = f_{pu}^{-1}(cl(f_{pu}(A,E))$.

Thus, $f_{pu}(cl^*(A,E)) \tilde{\subseteq} cl(f_{pu}(A,E))$.□

Remark (3.3.24):

$f_{pu}(cl^*(A,E)) \neq cl(f_{pu}(A,E))$.

Example :

Let X={a,b,c} ,E = {e$_1$, e$_2$} , Y={h$_1$, h$_2$, h$_3$}, K = {k$_1$, k$_2$}, $I = \{\phi_E\}$, and τ = { ϕ_E , X_E ,(F,E)}, ϑ = { ϕ_K , Y_K, (G,K)} be two soft topologies defined on X and Y respectively, (F,E)={(e_1,{ b }), (e_2,{ a,c })} , (G,K)={(k_1,{ h_1 }), (k_2,{ h_2,h_3 })} , define $p: E \to K$ *such that* $p(e_1) = k_1$, $p(e_2) = k_2$ and $u: X \to Y$ *such that* $u(a) = h_1, u(b) = h_3, u(c) = h_2$. Then , f_{pu} : (X,τ,E,I) → (Y, ϑ,K) is a soft mapping .

Since $f_{pu}^{-1}((G,K))= f_{pu}^{-1}(\{(k_1,\{h_3\}), (k_2,\{h_1,h_2\})\}) =\{(e_1,\{b\}), (e_2,\{a,c\})\}$ =(F,E) which is soft open set, so it is soft continuous mapping .

Now, let (A,E)=$\{(e_1, X), (e_2,\{a,b\})\}$ be a soft set in X . The SSIg-closed sets containing (A,E) are :

(A,E) and X_E . Then $cl^*(A,E)=$ (A,E) . We have $f_{pu}(A,E)=\{(k_1,Y), (k_2,\{h_1,h_3\})\}$. Then $cl(f_{pu}(A,E))=Y_K$, Hence, $f_{pu}(cl^*(A,E)) \tilde{\subseteq} Y_K = cl(f_{pu}(A,E))$, which mean that $cl(f_{pu}(A,E)) \tilde{\not\subset} (f_{pu}cl^*(A,E))$.□

Remark(3.3.25) :

$$f_{pu}(cl^*(A,E)) \tilde{\not\supseteq} cl(f_{pu}(A,E))$$

Example :

Let X={a,b} ,E = {e$_1$, e$_2$} , Y={h$_1$, h$_2$}, K = {k$_1$, k$_2$}, $I=\{\phi_E\}$ and τ = $\{\phi_E, X_E$,(F,E)}, $\vartheta = \{\phi_K, Y_K$, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)=$\{(e_1,\phi), (e_2,\{b\})\}$, (G,K)=$\{(k_1,Y), (k_2,\{h_1\})\}$, define $p:E\to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\to Y$ such that $u(a)=h_1, u(b)=h_2$. Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping. Since $f_{pu}^{-1}((G,K))= f_{pu}^{-1}(\{(k_1,Y),(k_2,\{h_1\})\}) =\{(e_1,X),(e_2,\{a\})\}$ which is not SSIg-open set. So it is not SSIg-continuous mapping .

On the other hand for each soft set (A,E) in SS(X,E) ,then $f_{pu}(cl^*(A,E)) \tilde{\subseteq} cl(f_{pu}(A,E))$.□

Proposition(3.3.26) :

Let f_{pu} : (X,τ,E,I) →(Y,ϑ,K) be a SSIg-continuous mapping. If (A,E) is any soft set in X ,then $f_{pu}^{-1}(int(A,E) \tilde{\subseteq} int^*(f_{pu}^{-1}(A,E))$ **.**

Proof :

Let (A,K) is any soft set in (Y,ϑ,K). Then *int(A,K)* is a soft open set in (Y,ϑ,K). But f_{pu} is SSIg-continuous, so $f_{pu}^{-1}(int(A,K))$ is SSIg-open by

Theorem(2.2.8) we have $f_{pu}^{-1}(int(A,K))= int^{*} f_{pu}^{-1}(int(A,K))$. But $int(A,K)\tilde{\subseteq}$ (A,K), then $f_{pu}^{-1}(int(A,K))\tilde{\subseteq}$ $f_{pu}^{-1}(A,K)$. Hence, $f_{pu}^{-1}(int(A,K))\tilde{\subseteq}$ $int^{*} f_{pu}^{-1}(A,K)$. □

Remark(3.3.27):

In proposition(3.3.26), $f_{pu}^{-1}(int(A,E)\tilde{\not\supseteq} int^{*}(f_{pu}^{-1}(A,E))$ in general.

Example:

Let X={a,b} ,E = {e_1, e_2}, Y={d, c}, K = {k_1, k_2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E ,(F,E)}, ϑ = {ϕ_K , Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{b}), (e_2,ϕ)} , (G,K)={(k_1,{d}), (k_2,Y)} , define $p:E\to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\to Y$ *such that* $u(a)=d$,$u(b)=c$. Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping . Since $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$({(k_1,{d}), (k_2,Y)}) ={(e_1,{a}), (e_2,X)} which is not SSIg-open set. So it is not SSIg-continuous mapping.

On the other hand for each soft set (A,K) in SS(Y,K) ,then we get $f_{pu}^{-1}(int(A,K))$ $\tilde{\subseteq}$ $int^{*} f_{pu}^{-1}(A,K)$. □

Remark(3.3.28):

In Proposition(3.3.26), $f_{pu}^{-1}(int(A,E)\neq int^{*}(f_{pu}^{-1}(A,E))$ in general.

Example:

Let X={a,b} ,E = {e_1, e_2} , Y={d, c}, K = {k_1, k_2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E ,(F,E)}, ϑ = {ϕ_K , Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{a}), (e_2,X)} , (G,K)={(k_1,{d}), (k_2,Y)} , define $p:E\to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\to Y$ *such that* $u(a)=d$,$u(b)=c$. Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping . It is clear that f_{pu} is an SSIg-continuous mapping .

On the other hand let (A,K) be soft set in SS(X,E) such that (A,K) = {(k_1,{d}), (k_2,ϕ)}, then int(A,E) $=\phi_K$ so $f_{pu}^{-1}(int(A,K))=\phi_E$ and $f_{pu}^{-1}(A,K)=$

$\{(e_1,\{a\}),\ (e_2,\phi)\}$, so $\text{int}^* f_{pu}^{-1}(A,K)=f_{pu}^{-1}(A,K)=\{(e_1,\{a\}),\ (e_2,\phi)\}$, therefore $int^* f_{pu}^{-1}(A,K) \tilde{\not\subset} f_{pu}^{-1}(int(A,K))$. □

Proposition(3.3.29) :

Let f_{pu} : (X,τ,E,I) →(Y,ϑ,K) be a SSIg-continuous mapping. If (A,E) is any soft set in over X , then $\text{int}(f_{pu}(A,E)) \tilde{\subseteq} f_{pu}(\text{int}^*(A,E))$.

Proof :

Let (A,E) be any soft set over X . Then $f_{pu}(A,E)$ is a soft set over Y and $\text{int}(f_{pu}(A,E))$ is soft open set over Y. But f_{pu} is a SSIg-continuous mapping . Therefore $f_{pu}^{-1}(\text{int}(f_{pu}(A,E)))$ is a SSIg-open set over X. Then $\text{int}^*(f_{pu}^{-1}(\text{int}(f_{pu}(A,E)))=f_{pu}^{-1}(\text{int}(f_{pu}(A,E)))$ by Theorem(2.2.8).

$\text{int}^*(f_{pu}^{-1}(\text{int}(f_{pu}(A,E))) \tilde{\subseteq} \text{int}^*(f_{pu}^{-1}(f_{pu}(A,E))) \subseteq \text{int}^*(A,E)$, therefore $f_{pu}^{-1}(\text{int}(f_{pu}(A,E)))$ $\tilde{\subseteq} \text{int}^*(A,E)$. Thus $\text{int}(f_{pu}(A,E)) \tilde{\subseteq} f_{pu}(\text{int}^*(A,E))$.□

Remark(3.3.30):

$\text{int}(f_{pu}(A,E)) \tilde{\not\supseteq} f_{pu}(\text{int}^*(A,E))$ in general.

Example :

Let X={a,b} ,E = {e_1, e_2} , Y={d, c}, K = {k_1, k_2}, $I=\{\phi_E\}$, and τ = $\{\phi_E,\ X_E$,(F,E)}, ϑ = { ϕ_K , Y_K, (G,K)} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{b}), (e_2,ϕ)} , (G,K)={(k_1,{d}), (k_2,Y)} , define $p:E \to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X \to Y$ *such that* $u(a)=d$,$u(b)=c$. Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping . Since $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$({(k_1,{d}), (k_2,Y)}) ={(e_1,{a}), (e_2,X)} which is not SSIg-open set. So it is not SSIg-continuous mapping .

On the other hand for each soft set (A,E) in SS(X,E) ,then $\text{int}(f_{pu}(A,E)) \tilde{\subseteq} f_{pu}(\text{int}^*(A,E))$.□

Remark(3.3.31):

$\text{int}(f_{pu}(A,E)) \neq f_{pu}(\text{int}^*(A,E))$ in general.

Example :

Let X={a,b}, E = {e_1, e_2}, Y={d, c}, K = {k_1, k_2}, $I=\{\phi_E\}$ and τ = $\{\phi_E, X_E$,(F,E)}, ϑ = $\{\phi_K, Y_K$, (G,K)} be two soft topologies defined on X and Y respectively, where (F,E)={(e_1,{a}),(e_2,X)}, (G,K)={(k_1,{d}),(k_2,Y)} , define $p:E \to K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X \to Y$ *such that* $u(a)=d$,$u(b)=c$. Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping . It is clear that f_{pu} is an SSIg-continuous mapping .

On the other hand let (A,E) be soft set in SS(X,E) such that (A,E) = {(e_1,{a}), (e_2,{b})}, then int^*(A,E) ={(e_1,{a}), (e_2,{b})}, so $f_{pu}(int^*(A,E))=$ {(k_1,{d}), (k_2,{c})} and $f_{pu}(A,E)$={(k_1,{d}), (k_2,{c})}, so $\text{int} f_{pu}(A,E)=\phi_K$, therefore $f_{pu}(\text{int}^*(A,E)) \tilde{\subseteq} \text{int}(f_{pu}(A,E))$.□

Definition(3.3.32) :

Let (X,τ_X,*A*,*I*) and (Y,τ_Y,B) be two soft topological spaces. Let f_{pu} : (X, τ_X,A) → (Y, τ_Y ,B) be a mapping. If f_{pu}^{-1}((G,B)) is a SSIg-open set over X for each SSIg-open set (G,B) over Y, then f_{pu} is said to be SSIg-irresolute mapping.

Proposition(3.3.33) :

Every SSIg-irresolute mapping is an SSIg-continuous mapping.

Proof:

Let (X,τ_X,*A*,*I*) and (Y,τ_Y,B) be two soft topological spaces. Let f_{pu} : (X, τ_X,A) → (Y, τ_Y ,B) be an SSIg- irresolute mapping. To show that f_{pu} SSIg-continuous. Let (G,B) be a soft open set over Y. Then by

Corollary(2.1.4), (G,B) is an SSIg-open set over Y. Since f_{pu} is an SSIg-irresolute. Then f_{pu}^{-1}((G,B)) is a SSIg-open set over X. Therefore, f_{pu} is an SSIg-continuous mapping. □

Remark(3.3.34) :

The SSIg-continuous mapping need not be SSIg-irresolute mapping in general.

Example :

Let X={a,b} ,E = {e_1, e_2} , Y={d, c}, K = {k_1, k_2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E ,(F,E)}, ϑ = {ϕ_K, Y_K} be two soft topologies defined on X and Y respectively , where (F,E)={(e_1,{b}),(e_2,ϕ)} define $p:E\rightarrow K$ *such that* $p(e_1)=k_1$, $p(e_2)=k_2$ and $u:X\rightarrow Y$ *such that* $u(a)=d$,$u(b)=c$.

Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping . It is clear that f_{pu} is an SSIg-continuous mapping. But f_{pu} is not SSIg- irresolute mapping since *(G,K)=* {(k_1,{d}), (k_2,Y)} is an SSIg-open set over Y, but $f_{pu}^{-1}((G,K))=f_{pu}^{-1}$({(k_1,{d}), (k_2,Y)}) = {(e_1,{a}), (e_2,X)} which is not SSIg-open set over X. □

Remark(3.3.35) :

The notions SSIg-irresolute mapping and soft continuous mapping are independent. We observe that in Example of Remark(3.3.34), that f_{pu} is SSIg-continuous mapping but f_{pu} is not SSIg- irresolute mapping. Also in the following example it is shown that f_{pu} is SSIg- irresolute mapping but f_{pu} is not SSIg- continuous mapping.

Example :

Let X={a,b} ,E = {e_1, e_2}, Y={d, c}, K = {k_1, k_2}, $I=\{\phi_E\}$ and τ = {ϕ_E, X_E }, ϑ = {ϕ_K, Y_K ,*(G,K)*} be two soft topologies defined on X and Y respectively , where *(G,K)*= {(k_1,{d}), (k_2,Y)} define $p: E \to K$ *such that* $p(e_1) = k_1$, $p(e_2) = k_2$ and $u: X \to Y$ *such that* $u(a) = d$,$u(b) = c$.

Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is a soft mapping . Since $f_{pu}^{-1}((G,K)) = f_{pu}^{-1}$ ({(k_1,{d}), (k_2,Y)}) = {(e_1,{a}), (e_2,X)} which is not SSIg-open set over X. Therefore f_{pu} is not SSIg-continuous mapping. But f_{pu} is an SSIg- irresolute mapping , since the inverse image under f_{pu} for every SSIg-open set, (SSIg-closed set),over Y is an SSIg-open set, (SSIg-closed set) . □

Definition(3.3.36) :

Let (X,τ_X,*A*,I) and (Y,τ_Y,B) be two soft topological spaces. Let f_{pu} : (X, τ_X,A) → (Y, τ_Y ,B) be a mapping. Then f_{pu} is said to be SSIg-homeomorphism mapping if it is bijective, SSIg-open(SSIg-closed) and SSIg-continuous.

Remark(3.3.37):

We say that (X,τ_X,*A*,I) is an SSIg-homeomorphic to (Y,τ_Y,B) if there exists f_{pu} : (X, τ_X,A) → (Y, τ_Y ,B) is an SSIg-homeomorphism and denoted by (X,τ_X,*A*,I) $\overset{SSIg}{\cong}$ (Y,τ_Y,B) .

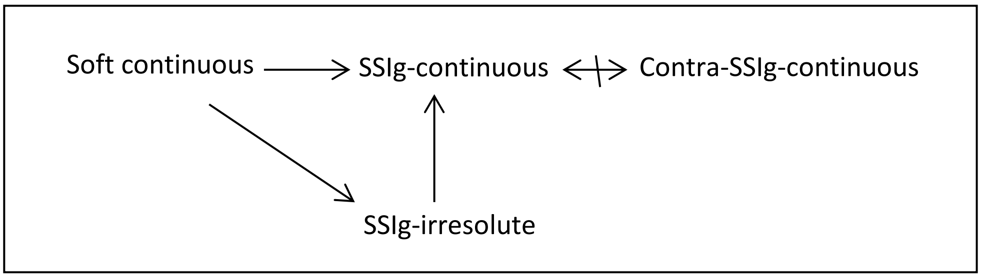
Example(3.3.38)

Let X={a,b} ,E = {e_1, e_2}, Y={d, c}, K = {k_1, k_2}, $I = \{\phi_E\}$ and τ = {ϕ_E, X_E},ϑ= {ϕ_K, Y_K} be two soft topologies defined on X and Y respectively, define $p: E \to K$ *such that* $p(e_1) = k_1$, $p(e_2) = k_2$ and $u: X \to Y$ *such that* $u(a) = d$,$u(b) = c$.

Then , f_{pu} : (X,τ,E,I) →(Y,ϑ,K) is an SSIg-homeomorphism. □

Note(3.3.39):

In the following diagram we discuss the relation between the kinds of soft mappings



Future work : In the future we can use the SSIg-open set to define SSIg-compact (connected, paracompact…etc), also we can define SSIg-T_i where i=1,2,3,4. On the other hand we can discuss the relation between SSIg-open set with other types of soft open sets in a soft topology.

References

[1] Georgiou D. N., A. C. Megaritis, **Soft set theory and topology**, Appl. Gen. Topol. 15, 93-109(2014).

[2] Georgiou D. N., A. C. Megaritis2 and V. I. Petropoulos1, **On soft topological spaces**, Applied Mathematics & Information Sciences 7, No. 5, 1889-1901 (2013).

[3] Hussain S., Ahmad B., **Some properties of soft topological spaces**, Comput. Math. Appl., **62**, 4058-4067 (2011).

[4] Hamlett T. R., Jankovic D., "**Ideals in general topology**", General Topology and Applications, pp. 115–125(1988).

[5] Jafari S., Rajesh N., **Generalized Closed Sets with Respect to an Ideal**, European Journal of Pure and Applied Mathematics,4, 147-151(2011).

[6] Kandil A., Tantawy O. A. E., El-Sheikh S. A. and A. M. Abd El-latif, Supra **Generalized Closed Soft Sets with Respect to an Soft Ideal in Supra Soft Topological Spaces**, Applied Mathematics & Information Sciences 8, No. 4, 1731-1740 (2014).

[7] Kandil A., Tantawy O. A. E., El-Sheikh S. A., Abd El-latif A. M., **Soft Ideal Theory Soft Local Function and Generated Soft Topological Spaces**, Applied Mathematics & Information Sciences 8, No. 4, 1595-1603 (2014).

[8] Kannan K., **Soft generalized closed sets in soft topological spaces** , Journal of Theoretical and Applied Information Technology, 37,17-21 (2012).

[9] Levine N., **Generalized closed sets in topology**, Rend. Circ. Mat. Palermo(2), 19, 89 – 96, (1970).

[10] Molodtsov D. A., **Soft set theory-first results**, Computers and Mathematics with Applications, **37,4**, 19-31 (1999).

[11] Mustafa H. I., Sleim F. M., **Soft Generalized Closed Sets with Respect to an Ideal in Soft Topological Spaces**, Applied Mathematics & Information Sciences, 8,2, 665-671 (2014).

[12] Maji P.K., Biswas *R., Roy A.R.*, **Soft set theory**. Computers and Mathematics with Applications. 45, 555-562 (2003).

[13] Maji P.K., Biswas R., Roy, A.R. **An application of soft sets in a decisions making problems**, Computers and Mathematics with Applications, 44, 1077-1083(2002).

[14] Majumdar P., Samanta S.K., **On Soft Mappings**, Computers and Mathematics with Applications, 60 , 2666–2672(2010).

[15] Maragathavalli S., Sathiyavathi, **Strongly Generalized closed sets in Ideal Topological Spaces**, International Journal of Physics and Mathematical Research 3, No. 2, 011-013 (2014).

[16] Nandhini1 T., Kalaichelvi A., **Soft ĝ-Closed Sets in Soft Topological Spaces**, International Journal of Innovative Research in Science, Engineering and Technology, 3 , 14595-14600(2014).

[17] Peyghan E., Samadi B. , Tayebi A., **About Soft Topological Spaces**, Journal of new results of sciences 2, 60-75 (2013).

[18] Rajesh K., Bobin G., **Soft Graphs**, Gen. Math. Notes, 21,2, 75-86 (2014).

[19] Shastri N., **Textbook of Topology**, S. Chand & Company LTD,(p.102),1980.

[20] Singh D. , Onyeozili I. A. **On some new properties of soft set operations**, International Journal of Computer Applications (0975- 8887), 59(4), 39- 44 (2012).

[21] Zorlutuna I., Akdag M., Min W. K., Atmaca S., **Remarks on soft topological spaces**, Annals of Fuzzy Mathematics and Informatics 3, no. 2, 171–185(2012).

[22] Zhao X**.,** Feng F., Young Jun B., **Soft semirings**, Computers and Mathematics with Applications 56,2621–2628(2008).

المستخلص

في هذا العمل، نقدم وندرس نوعا جديدا من المجاميع الناعمة المغلقة من النمط SSIg في الفضاء التبولوجي الناعم مع مثالي، والتي أسميناها المجموعة الناعمة المعممة المغلقة بقوة المتعلقة بمثالي I ،وهي المجموعة الناعمة (A,E) في الفضاء التبولوجي الناعم (X,τ,E) مع مثالي I ، حيث $cl(int(A,E))-(B,E)\in I$ طالما $(A,E)\tilde{\subseteq}(B,E)$ و ان (B,E) مجموعه ناعمه مفتوحه. وسنرمز لهذه المجموعة بالرمز SSIg-closed . المتممة للمجموعة الناعمة المعممة المغلقة بقوة المتعلقة بمثالي I هي المجموعة الناعمة المعممة المفتوحة بقوة المتعلقة بمثالي I .

درسنا الخصائص لــ SSIg-closed و استخدمنا SSIg-open لتعريف خمسة انواع لمجاميع مشتقه وهي داخل و انغلاق و اشتقاق و الحد و كذلك حدود للمجموعة الناعمه من النمط SSIg مع العلاقات والخصائص .

على الجانب الآخر، عرفنا أنواعا جديدة من التطبيقات الناعمة بين الفضاءات التبولوجيه الناعمة مثل المستمرة و عكس المستمرة و المفتوحة و المغلقة وكذلك المتحيرة للمجموعة الناعمة من النمط SSIg ، درسنا العلاقات بين هذه الأنواع من التطبيقات و تكوين تركيب بين اثنين من التطبيقات من نفس النوع او من نوعين مختلفين، مع البراهين أو أمثلة مضادة.

جمهورية العراق

وزارة التعليم العالي والبحث العلمي

جامعة بغداد

كلية التربية للعلوم الصرفة / ابن الهيثم

المجموعة المغلقة من النمط SSIg في فضاء تبولوجي ناعم بالنسبة الى مثالي

رسالة

مقدمه الى كلية التربية للعلوم الصرفة / ابن الهيثم – جامعة بغداد وهي جزء من متطلبات نيل درجة الماجستير في علوم الرياضيات

من قبل

اليسع جاسم بديوي

بأشراف

أ.م.د. نرجس عبد الجبار

جمادي الاول 143   اذار، 20